Quadratic Equations

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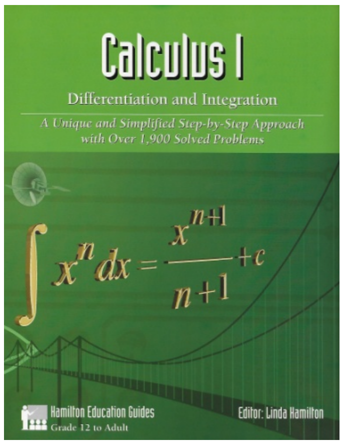

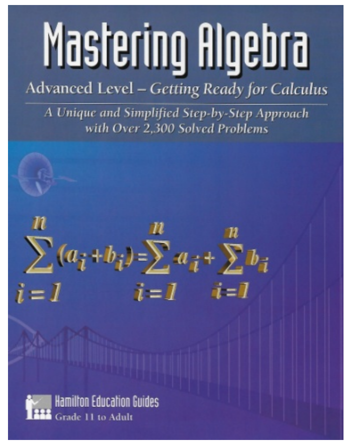

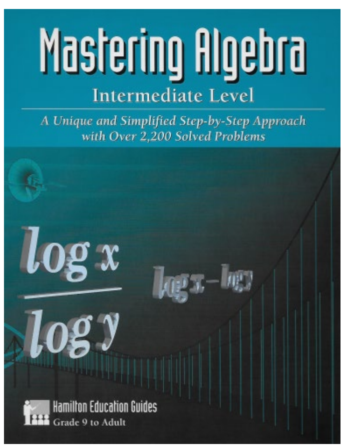

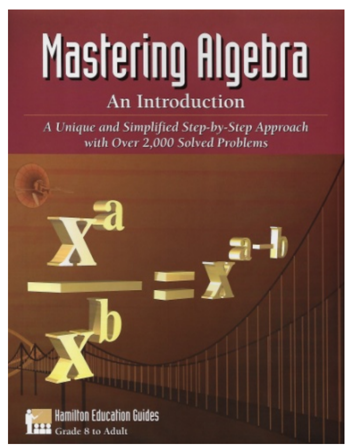

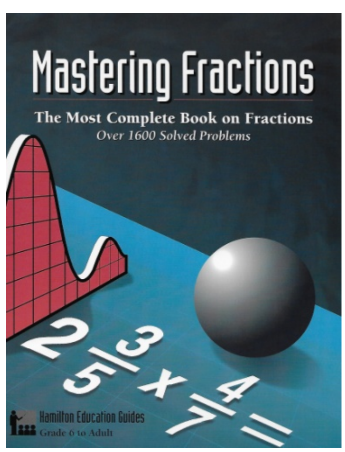

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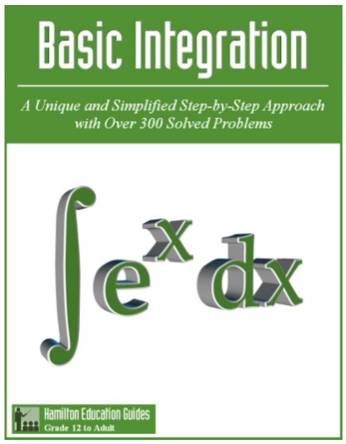

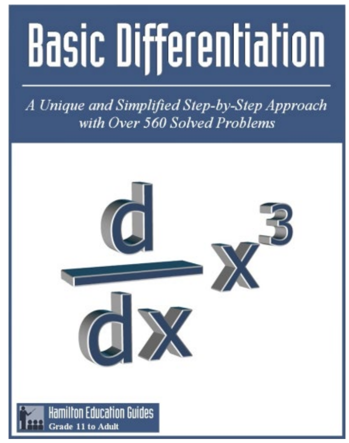

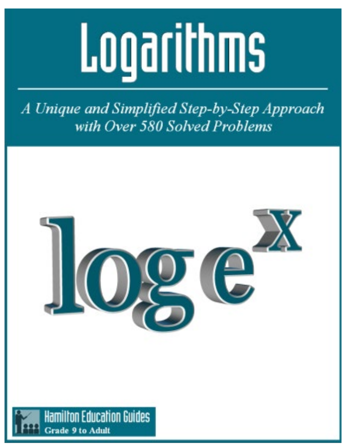

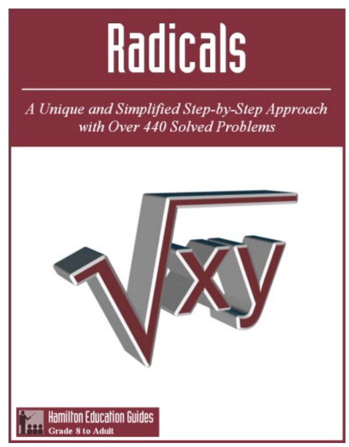

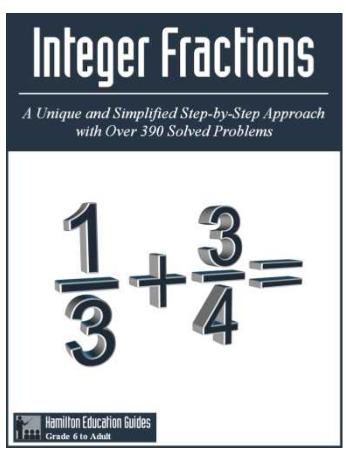

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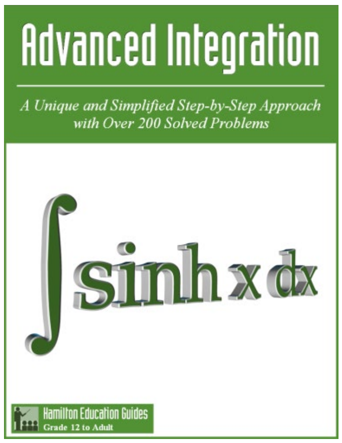

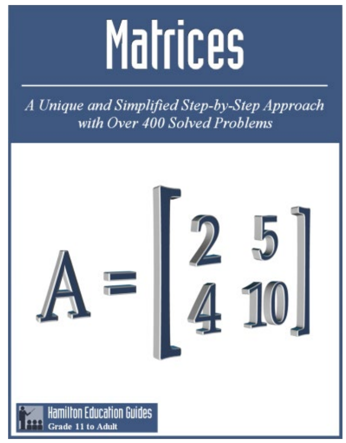

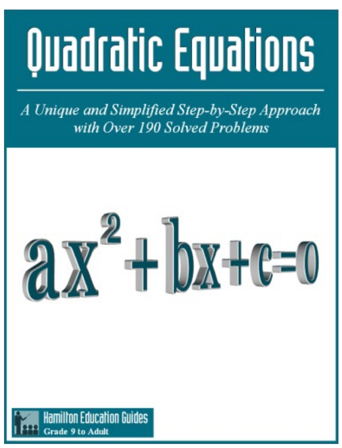

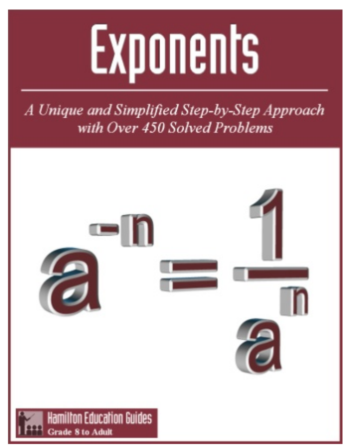

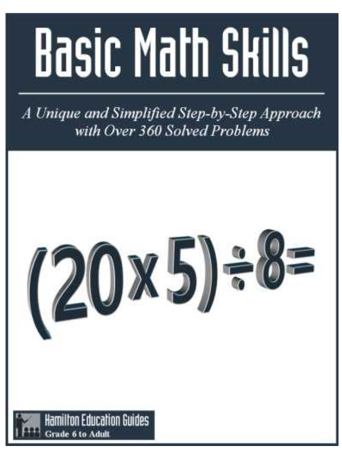

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Acknowledgments

The primary motivating factor in writing the Hamilton Education Guides manual series is to provide students with specific subjects on mathematics. I am grateful to John Hamilton for his editorial comments, cover design, and suggestions on easier presentation of the topics. I would also like to acknowledge the original contributors of the Hamilton Education Guides books for their editorial reviews. Finally, I would like to thank my family for their understanding and patience in allowing me to prepare this manual.

Introduction and Overview

It is my belief that the key to learning mathematics is through positive motivation. Students can be greatly motivated if subjects are presented concisely and the problems are solved in a detailed step by step approach. This keeps students motivated and provides a great deal of encouragement in wanting to learn the next subject or to solve the next problem. During my teaching career, I found this method to be an effective way of teaching. I hope by presenting equations in this format, more students will become interested in the subject of mathematics.

This manual is a chapter from my Mastering Algebra – Intermediate Level book with the primary focus on the subject of quadratic equations. The scope of this manual is intended for educational levels ranging from the 9th grade to adult. The manual can also be used by students in home study programs, parents, teachers, special education programs, preparatory schools, and adult educational programs including colleges and universities as a supplementary manual. A fundamental understanding of basic mathematical operations such as addition, subtraction, multiplication, and division is required.

This manual addresses quadratic equations and how they are simplified and mathematically operated. Students learn how to solve quadratic equations using techniques such as factoring, the Quadratic Formula method, the Square Root Property method, and the Completing-the-Square method. Detailed solutions to the exercises are provided in the Appendix. Students are encouraged to solve each problem in the same detail and step by step format as shown in the text.

It is my hope that all Hamilton Education Guides books and manuals stand apart in their understandable treatment of the presented subjects and for their clarity and special attention to detail. I hope readers of this manual will find it useful.

With best wishes,

Dan Hamilton

Quadratic Equations

Quick Reference to Case Problems

Quadratic Equations

The objective of this manual is to improve the student's ability to solve quadtratic equations and provide factoring methods involving quadratic equations. Quadratic equations and the quadratic formula are introduced in Section 1.1. Solving different forms of quadratic equations using the quadratic formula is introduced in Section 1.2. Steps as to how quadratic equations are solved using the Square Root Property method are addressed in Section 1.3. Solving quadratic equations by Completing-the-Square method are addressed in Section 1.4. In Section 1.5, solving quadratic equations containing radicals and fractions are discussed. Choosing the most suitable method in factoring polynomials and solving second degree equations is discussed in Section 1.6. Cases presented in each section are concluded by solving additional examples with practice problems to further enhance the students ability. Students are encouraged to gain a thorough knowledge on the different factoring polynomials and solution methods introduced in Chapter 3 of the Mastering Algebra – Intermediate Level book. Knowing how to factor polynomials and solve quadratic equations will greatly improve the student's ability in solving more advanced math concepts.

1.1 Quadratic Equations and the Quadratic Formula

A quadratic equation is an equation in which the highest power of the variable is 2. For example, $3x^2 - 16x + 5 = 0$, $x^2 = 16$, $w^2 + 9w = 0$, $x^2 - 4x + 3 = 0$, $x^2 = -11x - 24$, and $y^2 - 4 = 0$ are all examples of quadratic equations. Note that any equation that can be written in the form of $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, is called a **quadratic equation**. A quadratic equation represented in the form of $ax^2 + bx + c = 0$ is said to be in its **standard form**. In the following sections we will learn how to solve and represent the solutions to quadratic equations in factored form. However, in order to solve any quadratic equation we first need to become familiar with the quadratic formula.

The Quadratic Formula

To derive the quadratic formula we start with the standard quadratic equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers and use the method of completing the square to solve the equation as follows:

Step 1 Add $-c$ to both sides of the equation.

$$ax^2 + bx + c - c = -c \;;\; ax^2 + bx = -c$$

Step 2 Divide both sides of the equation by a.

$$\frac{ax^2}{a} + \frac{bx}{a} = -\frac{c}{a} \;;\; x^2 + \frac{bx}{a} = -\frac{c}{a}$$

Step 3 Divide $\frac{b}{a}$, the coefficient of x, by 2 and square the term to obtain $\left(\frac{b}{2a}\right)^2$. Add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 4 Write the left hand side of the equation , which is a perfect square trinomial, in its equivalent square form.

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

Step 5 Simplify the right hand side of the equation using the fraction techniques.

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2\;;\;\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}\;;\;\left(x+\frac{b}{2a}\right)^2=\frac{\left(4a^2\cdot -c\right)+\left(a\cdot b^2\right)}{4a^2\cdot a}$$

$$;\;\left(x+\frac{b}{2a}\right)^2=\frac{ab^2-4a^2c}{4a^3}\;;\;\left(x+\frac{b}{2a}\right)^2=\frac{a\left(b^2-4ac\right)}{4a^3}\;;\;\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Step 6 Take the square root of both sides of the equation.

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}\;;\;x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{\sqrt{2^2a^2}}\;;\;x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

Step 7 Solve for x by adding $-\frac{b}{2a}$ to both sides of the equation.

$$x+\frac{b}{2a}-\frac{b}{2a}=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}\;;\;x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}\;;\;x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The equation $\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$ is referred to as the **quadratic formula**. Note that the quadratic formula has two solutions $x=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x=\frac{-b-\sqrt{b^2-4ac}}{2a}$. We use these solutions to write the quadratic equation $ax^2+bx+c=0$ in its equivalent factored form, i.e.,

$$\boldsymbol{ax^2+bx+c=0}\text{ is factorable to }\boldsymbol{\left(x-\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b+\sqrt{b^2-4ac}}{2a}\right)=0}$$

Let's check the above factored product using the FOIL method. The result should be equal to $ax^2+bx+c=0$.

Check:

$$\left(x-\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b+\sqrt{b^2-4ac}}{2a}\right)=0\;;\;\left(x+\frac{b-\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b+\sqrt{b^2-4ac}}{2a}\right)=0$$

$$;\;x\cdot x+\left(\frac{b+\sqrt{b^2-4ac}}{2a}\right)\cdot x+\left(\frac{b-\sqrt{b^2-4ac}}{2a}\right)\cdot x+\left(\frac{b+\sqrt{b^2-4ac}}{2a}\right)\cdot\left(\frac{b-\sqrt{b^2-4ac}}{2a}\right)=0$$

$$;\;x^2+\left(\frac{b+\sqrt{b^2-4ac}}{2a}+\frac{b-\sqrt{b^2-4ac}}{2a}\right)x+\left(\frac{\left(b+\sqrt{b^2-4ac}\right)\left(b-\sqrt{b^2-4ac}\right)}{2a\cdot 2a}\right)=0$$

$$;\;x^2+\left(\frac{b+\sqrt{b^2-4ac}+b-\sqrt{b^2-4ac}}{2a}\right)x+\left(\frac{\left(b^2-b\sqrt{b^2-4ac}+b\sqrt{b^2-4ac}-\sqrt{b^2-4ac}\cdot\sqrt{b^2-4ac}\right)}{4a^2}\right)=0$$

; $x^2+\left(\frac{b+b}{2a}\right)x+\left[\frac{b^2-\left(b^2-4ac\right)}{4a^2}\right]=0$; $x^2+\left(\frac{2b}{2a}\right)x+\left(\frac{b^2-b^2+4ac}{4a^2}\right)=0$; $x^2+\frac{b}{a}x+\frac{4ac}{4a^2}=0$

; $x^2+\frac{b}{a}x+\frac{c}{a}=0$; $\frac{x^2}{1}+\frac{bx}{a}+\frac{c}{a}=0$; $\frac{ax^2+bx+c}{a}=0$; $\frac{ax^2+bx+c}{a}=\frac{0}{1}$; $\left(ax^2+bx+c\right)\cdot 1=a\cdot 0$ which is the same as $ax^2+bx+c=0$.

The quadratic formula is a powerful formula and should be memorized. In the following sections we will use this formula to solve different types of quadratic equations.

Practice Problems - Quadratic Equations and Quadratic Formula

Section 1.1 Practice Problems - Given the following quadratic equations identify the coefficients a, b, and c.

1. $3x=-5+2x^2$
2. $2x^2=5$
3. $3w^2-5w=2$
4. $15=-y^2-3$
5. $x^2+3=5x$
6. $-u^2+2=3u$
7. $y^2+5y-2=0$
8. $-3x^2=2x-1$
9. $p^2=p-1$
10. $3x-2=x^2$

1.2 Solving Quadratic Equations Using the Quadratic Formula

As was stated earlier, the quadratic formula can be used to solve any quadratic equation by expressing the equation in the standard form of $ax^2+bx+c=0$ and by substituting the equivalent numbers for a, b, and c into the quadratic formula. In this section we will learn how to solve quadratic equations of the form $ax^2+bx+c=0$, where $a=1$ (Case I) and where $a \rangle 1$ (Case II), using the quadratic formula.

Case I Solving Quadratic Equations of the Form $ax^2+bx+c=0$, where $a=1$, Using the Quadratic Formula

Quadratic equations of the form $ax^2+bx+c=0$, where $a=1$, are solved using the following steps:

Step 1 Write the equation in standard form.

Step 2 Identify the coefficients a, b, and c.

Step 3 Substitute the values for a, b, and c into the quadratic equation $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. Simplify the equation.

Step 4 Solve for the values of x. Check the answers by either substituting the x values into the original equation or by multiplying the factored product using the FOIL method.

Step 5 Write the quadratic equation in its factored form.

Examples with Steps

The following examples show the steps as to how quadratic equations are solved using the quadratic formula:

Example 1.2-1

Solve the quadratic equation $x^2+5x=-4$.

Solution:

Step 1 $\boxed{x^2+5x=-4}$; $\boxed{x^2+5x+4=-4+4}$; $\boxed{x^2+5x+4=0}$

Step 2 Let: $\boxed{a=1}$, $\boxed{b=5}$, and $\boxed{c=4}$. Then,

Step 3 Given: $\boxed{x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\dfrac{-5\pm\sqrt{5^2-4\times1\times4}}{2\times1}}$; $\boxed{x=\dfrac{-5\pm\sqrt{25-16}}{2}}$

; $\boxed{x=\dfrac{-5\pm\sqrt{9}}{2}}$; $\boxed{x=\dfrac{-5\pm\sqrt{3^2}}{2}}$; $\boxed{x=\dfrac{-5\pm3}{2}}$

Step 4 Separate $x=\dfrac{-5\pm3}{2}$ into two equations.

I. $\boxed{x=\dfrac{-5+3}{2}}$; $\boxed{x=-\dfrac{\cancel{2}}{\cancel{2}}}$; $\boxed{x=-\dfrac{1}{1}}$; $\boxed{\mathbf{x=-1}}$

II. $\boxed{x=\frac{-5-3}{2}}$; $\boxed{x=-\frac{\overset{4}{\cancel{8}}}{\cancel{2}}}$; $\boxed{x=-\frac{4}{1}}$; $\boxed{\boldsymbol{x=-4}}$

Check No. 1: I. *Let* $x=-1$ *in* $x^2+5x=-4$; $(-1)^2+(5\times-1)\overset{?}{=}-4$; $1-5\overset{?}{=}-4$; $-4=-4$

II. *Let* $x=-4$ *in* $x^2+5x=-4$; $(-4)^2+(5\times-4)\overset{?}{=}-4$; $16-20\overset{?}{=}-4$; $-4=-4$

Check No. 2: $x^2+5x+4\overset{?}{=}(x+1)(x+4)$; $x^2+5x+4\overset{?}{=}(x\cdot x)+(4\cdot x)+(1\cdot x)+(1\cdot 4)$

; $x^2+5x+4\overset{?}{=}x^2+4x+x+4$; $x^2+5x+4\overset{?}{=}x^2+(4+1)x+4$; $x^2+5x+4=x^2+5x+4$

Step 5 Therefore, the equation $x^2+5x+4=0$ can be factored to $\boldsymbol{(x+1)(x+4)=0}$.

Example 1.2-2

Solve the quadratic equation $x^2=-12x-35$.

Solution:

Step 1 $\boxed{x^2=-12x-35}$; $\boxed{x^2+12x=-12x+12x-35}$; $\boxed{x^2+12x=0-35}$; $\boxed{x^2+12x=-35}$

; $\boxed{x^2+12x+35=-35+35}$; $\boxed{x^2+12x+35=0}$

Step 2 Let: $\boxed{a=1}$, $\boxed{b=12}$, and $\boxed{c=35}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-12\pm\sqrt{12^2-4\times1\times35}}{2\times1}}$; $\boxed{x=\frac{-12\pm\sqrt{144-140}}{2}}$

; $\boxed{x=\frac{-12\pm\sqrt{4}}{2}}$; $\boxed{x=\frac{-12\pm\sqrt{2^2}}{2}}$; $\boxed{x=\frac{-12\pm2}{2}}$

Step 4 Separate $x=\frac{-12\pm2}{2}$ into two equations.

I. $\boxed{x=\frac{-12+2}{2}}$; $\boxed{x=-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}}$; $\boxed{x=-\frac{5}{1}}$; $\boxed{\boldsymbol{x=-5}}$

II. $\boxed{x=\frac{-12-2}{2}}$; $\boxed{x=-\frac{\overset{7}{\cancel{14}}}{\cancel{2}}}$; $\boxed{x=-\frac{7}{1}}$; $\boxed{\boldsymbol{x=-7}}$

Check No. 1: I. *Let* $x=-5$ *in* $x^2=-12x-35$; $(-5)^2\overset{?}{=}(-12\times-5)-35$; $25\overset{?}{=}60-35$; $25=25$

II. *Let* $x=-7$ *in* $x^2=-12x-35$; $(-7)^2\overset{?}{=}(-12\times-7)-35$; $49\overset{?}{=}84-35$; $49=49$

Check No. 2: $x^2+12x+35\overset{?}{=}(x+5)(x+7)$; $x^2+12x+35\overset{?}{=}(x\cdot x)+(7\cdot x)+(5\cdot x)+(5\cdot 7)$

; $x^2+12x+35\overset{?}{=}x^2+7x+5x+35$; $x^2+12x+35\overset{?}{=}x^2+(7+5)x+35$

; $x^2+12x+35=x^2+12x+35$

Step 5 Therefore, the equation $x^2+12x+35=0$ can be factored to $\boldsymbol{(x+5)(x+7)=0}$.

Example 1.2-3

Solve the quadratic equation $x^2 - 5x + 6 = 0$.

Solution:

Step 1 $\boxed{\textit{Not Applicable}}$

Step 2 Let: $\boxed{a = 1}$, $\boxed{b = -5}$, and $\boxed{c = 6}$. Then,

Step 3 Given: $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}}$; $\boxed{x = \dfrac{5 \pm \sqrt{25 - 24}}{2}}$

; $\boxed{x = \dfrac{5 \pm \sqrt{1}}{2}}$; $\boxed{x = \dfrac{5 \pm 1}{2}}$

Step 4 Separate $x = \dfrac{5 \pm 1}{2}$ into two equations.

I. $\boxed{x = \dfrac{5 + 1}{2}}$; $\boxed{x = \dfrac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{x = \dfrac{3}{1}}$; $\boxed{\boldsymbol{x = 3}}$

II. $\boxed{x = \dfrac{5 - 1}{2}}$; $\boxed{x = \dfrac{\overset{2}{\cancel{4}}}{\cancel{2}}}$; $\boxed{x = \dfrac{2}{1}}$; $\boxed{\boldsymbol{x = 2}}$

Check No. 1: I. *Let* $x = 3$ *in* $x^2 - 5x + 6 = 0$; $(3)^2 + (-5 \times 3) + 6 \overset{?}{=} 0$; $9 - 15 + 6 \overset{?}{=} 0$; $15 - 15 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = 2$ *in* $x^2 - 5x + 6 = 0$; $(2)^2 + (-5 \times 2) + 6 \overset{?}{=} 0$; $4 - 10 + 6 \overset{?}{=} 0$; $4 - 4 \overset{?}{=} 0$; $0 = 0$

Check No. 2: $x^2 - 5x + 6 \overset{?}{=} (x - 3)(x - 2)$; $x^2 - 5x + 6 \overset{?}{=} (x \cdot x) + (-2 \cdot x) + (-3 \cdot x) + (-3 \cdot -2)$; $x^2 - 5x + 6 \overset{?}{=} x^2 - 2x - 3x + 6$; $x^2 - 5x + 6 \overset{?}{=} x^2 + (-2 - 3)x + 6$; $x^2 - 5x + 6 = x^2 - 5x + 6$

Step 5 Therefore, the equation $x^2 - 5x + 6 = 0$ can be factored to $\boldsymbol{(x - 3)(x - 2) = 0}$.

Example 1.2-4

Solve the quadratic equation $x^2 + 1 = -2x$.

Solution:

Step 1 $\boxed{x^2 + 1 = -2x}$; $\boxed{x^2 - 2x + 1 = -2x + 2x}$; $\boxed{x^2 + 2x + 1 = 0}$

Step 2 Let: $\boxed{a = 1}$, $\boxed{b = 2}$, and $\boxed{c = 1}$. Then,

Step 3 Given: $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 1}}{2 \times 1}}$; $\boxed{x = \dfrac{-2 \pm \sqrt{4 - 4}}{2}}$; $\boxed{x = \dfrac{-2 \pm 0}{2}}$

Step 4 Separate $x = \dfrac{-2 \pm 0}{2}$ into two equations.

I. $\boxed{x=\frac{-2+0}{2}}$; $\boxed{x=-\frac{\cancel{2}}{\cancel{2}}}$; $\boxed{x=-\frac{1}{1}}$; $\boxed{\boldsymbol{x=-1}}$

II. $\boxed{x=\frac{-2-0}{2}}$; $\boxed{x=-\frac{\cancel{2}}{\cancel{2}}}$; $\boxed{x=-\frac{1}{1}}$; $\boxed{\boldsymbol{x=-1}}$

Check No. 1: *Let* $x=-1$ *in* $x^2+1=-2x$; $(-1)^2+1\overset{?}{=}-2\times-1$; $1+1\overset{?}{=}2$; $2=2$

Check No. 2: $x^2+2x+1\overset{?}{=}(x+1)(x+1)$; $x^2+2x+1\overset{?}{=}(x\cdot x)+(1\cdot x)+(1\cdot x)+(1\cdot 1)$
; $x^2+2x+1\overset{?}{=}x^2+x+x+1$; $x^2+2x+1\overset{?}{=}x^2+(1+1)x+1$; $x^2+2x+1=x^2+2x+1$

Step 5 Thus, the equation $x^2+2x+1=0$ has two identical solutions and can be factored to $\boldsymbol{(x+1)(x+1)=0}$

Example 1.2-5

Solve the quadratic equation $7x=-x^2-2$.

Solution:

Step 1 $\boxed{7x=-x^2-2}$; $\boxed{+x^2+7x=-x^2+x^2-2}$; $\boxed{x^2+7x=0-2}$; $\boxed{x^2+7x=-2}$
; $\boxed{x^2+7x+2=-2+2}$; $\boxed{x^2+7x+2=0}$

Step 2 Let: $\boxed{a=1}$, $\boxed{b=7}$, and $\boxed{c=2}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-7\pm\sqrt{7^2-4\times1\times2}}{2\times1}}$; $\boxed{x=\frac{-7\pm\sqrt{49-8}}{2}}$
; $\boxed{x=\frac{-7\pm\sqrt{41}}{2}}$; $\boxed{x=\frac{-7\pm6.4}{2}}$

Step 4 Separate $x=\frac{-7\pm6.4}{2}$ into two equations.

I. $\boxed{x=\frac{-7+6.4}{2}}$; $\boxed{x=-\frac{0.6}{2}}$; $\boxed{\boldsymbol{x=-0.3}}$

II. $\boxed{x=\frac{-7-6.4}{2}}$; $\boxed{x=-\frac{13.4}{2}}$; $\boxed{\boldsymbol{x=-6.7}}$

Check No. 1: I. *Let* $x=-0.3$ *in* $7x=-x^2-2$; $7\times-0.3\overset{?}{=}-(-0.3)^2-2$; $-2.1\overset{?}{=}-0.09-2$; $-2.1=-2.1$

II. *Let* $x=-6.7$ *in* $7x=-x^2-2$; $7\times-6.7\overset{?}{=}-(-6.7)^2-2$; $-46.9\overset{?}{=}-44.9-2$
; $-46.9=-46.9$

Check No. 2: $x^2+7x+2\overset{?}{=}(x+0.3)(x+6.7)$; $x^2+7x+2\overset{?}{=}(x\cdot x)+(6.7\cdot x)+(0.3\cdot x)+(0.3\cdot 6.7)$
; $x^2+7x+2\overset{?}{=}x^2+6.7x+0.3x+2$; $x^2+7x+2\overset{?}{=}x^2+(6.7+0.3)x+2$

$; \; x^2 + 7x + 2 = x^2 + 7x + 2$

Step 5 Thus, the equation $x^2 + 7x + 2 = 0$ can be factored to $\mathbf{(x+0.3)(x+6.7)=0}$.

Additional Examples - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a = 1$, Using the Quadratic Formula

The following examples further illustrate how to solve quadratic equations using the quadratic formula:

Example 1.2-6

Solve the quadratic equation $x^2 = 16x - 55$.

Solution:

First, write the equation in standard form, i.e., $x^2 - 16x + 55 = 0$

Next, let: $\boxed{a=1}$, $\boxed{b=-16}$, and $\boxed{c=55}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \times 1 \times 55}}{2 \times 1}}$; $\boxed{x = \frac{16 \pm \sqrt{256 - 220}}{2}}$; $\boxed{x = \frac{16 \pm \sqrt{36}}{2}}$

; $\boxed{x = \frac{16 \pm \sqrt{6^2}}{2}}$; $\boxed{x = \frac{16 \pm 6}{2}}$ Therefore:

I. $\boxed{x = \frac{16+6}{2}}$; $\boxed{x = \frac{\overset{11}{\cancel{22}}}{\cancel{2}}}$; $\boxed{x = \frac{11}{1}}$; $\boxed{\mathbf{x = 11}}$ II. $\boxed{x = \frac{16-6}{2}}$; $\boxed{x = \frac{\overset{5}{\cancel{10}}}{\cancel{2}}}$; $\boxed{x = \frac{5}{1}}$; $\boxed{\mathbf{x = 5}}$

Check No. 1: I. *Let* $x = 11$ *in* $x^2 = 16x - 55$; $11^2 \overset{?}{=} 16 \times 11 - 55$; $121 \overset{?}{=} 176 - 55$; $121 = 121$

II. *Let* $x = 5$ *in* $x^2 = 16x - 55$; $5^2 \overset{?}{=} 16 \times 5 - 55$; $25 \overset{?}{=} 80 - 55$; $25 = 25$

Check No. 2: $x^2 - 16x + 55 \overset{?}{=} (x-11)(x-5)$; $x^2 - 16x + 55 \overset{?}{=} (x \cdot x) + (-5 \cdot x) + (-11 \cdot x) + (-11 \cdot -5)$

; $x^2 - 16x + 55 \overset{?}{=} x^2 - 5x - 11x + 55$; $x^2 - 16x + 55 \overset{?}{=} x^2 + (-5-11)x + 55$

; $x^2 - 16x + 55 = x^2 - 16x + 55$

Therefore, the equation $x^2 - 16x + 55 = 0$ can be factored to $\mathbf{(x-11)(x-5)=0}$.

Example 1.2-7

Solve the quadratic equation $x^2 = -9x + 36$.

Solution:

First, write the equation in standard form, i.e., $x^2 + 9x - 36 = 0$

Next, let: $\boxed{a=1}$, $\boxed{b=9}$, and $\boxed{c=-36}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times -36}}{2 \times 1}}$; $\boxed{x = \frac{-9 \pm \sqrt{81 + 144}}{2}}$; $\boxed{x = \frac{-9 \pm \sqrt{225}}{2}}$

; $\boxed{x = \frac{-9 \pm \sqrt{15^2}}{2}}$; $\boxed{x = \frac{-9 \pm 15}{2}}$ Therefore:

I. $\boxed{x = \frac{-9+15}{2}}$; $\boxed{x = \frac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{x = \frac{3}{1}}$; $\boxed{\mathbf{x = 3}}$ II. $\boxed{x = \frac{-9-15}{2}}$; $\boxed{x = -\frac{\overset{12}{\cancel{24}}}{\cancel{2}}}$; $\boxed{x = -\frac{12}{1}}$; $\boxed{\mathbf{x = -12}}$

and the solution set is $\{\mathbf{3}, -\mathbf{12}\}$.

Check No. 1: I. *Let* $x = 3$ *in* $x^2 = -9x + 36$; $3^2 \overset{?}{=} -9 \times 3 + 36$; $9 \overset{?}{=} -27 + 36$; $9 = 9$

II. *Let* $x = -12$ *in* $x^2 = -9x + 36$; $(-12)^2 \overset{?}{=} -9 \times -12 + 36$; $144 \overset{?}{=} 108 + 36$; $144 = 144$

Check No. 2: $x^2 + 9x - 36 \overset{?}{=} (x-3)(x+12)$; $x^2 + 9x - 36 \overset{?}{=} (x \cdot x) + (12 \cdot x) + (-3 \cdot x) + (-3 \cdot 12)$

; $x^2 + 9x - 36 \overset{?}{=} x^2 + 12x - 3x - 36$; $x^2 + 9x - 36 \overset{?}{=} x^2 + (12-3)x - 36$

; $x^2 + 9x - 36 = x^2 + 9x - 36$

Therefore, the equation $x^2 + 9x - 36 = 0$ can be factored to $(\mathbf{x - 3})(\mathbf{x + 12}) = \mathbf{0}$.

Example 1.2-8

Solve the quadratic equation $x^2 + 11x + 24 = 0$.

Solution:

The equation is already in standard form.

Let: $\boxed{a = 1}$, $\boxed{b = 11}$, and $\boxed{c = 24}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-11 \pm \sqrt{11^2 - 4 \times 1 \times 24}}{2 \times 1}}$; $\boxed{x = \frac{-11 \pm \sqrt{121 - 96}}{2}}$; $\boxed{x = \frac{-11 \pm \sqrt{25}}{2}}$

; $\boxed{x = \frac{-11 \pm \sqrt{5^2}}{2}}$; $\boxed{x = \frac{-11 \pm 5}{2}}$ Therefore:

I. $\boxed{x = \frac{-11+5}{2}}$; $\boxed{x = -\frac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{x = -\frac{3}{1}}$; $\boxed{\mathbf{x = -3}}$ II. $\boxed{x = \frac{-11-5}{2}}$; $\boxed{x = -\frac{\overset{8}{\cancel{16}}}{\cancel{2}}}$; $\boxed{x = -\frac{8}{1}}$; $\boxed{\mathbf{x = -8}}$

and the solution set is $\{-\mathbf{3}, -\mathbf{8}\}$.

Check No. 1: I. *Let* $x = -3$ *in* $x^2 + 11x + 24 = 0$; $(-3)^2 + 11 \times -3 + 24 \overset{?}{=} 0$; $9 - 33 + 24 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = -8$ *in* $x^2 + 11x + 24 = 0$; $(-8)^2 + 11 \times -8 + 24 \overset{?}{=} 0$; $64 - 88 + 24 \overset{?}{=} 0$; $0 = 0$

Check No. 2: $x^2 + 11x + 24 \overset{?}{=} (x+3)(x+8)$; $x^2 + 11x + 24 \overset{?}{=} (x \cdot x) + (8 \cdot x) + (3 \cdot x) + (3 \cdot 8)$

; $x^2 + 11x + 24 \overset{?}{=} x^2 + 8x + 3x + 24$; $x^2 + 11x + 24 \overset{?}{=} x^2 + (8+3)x + 24$

; $x^2 + 11x + 24 = x^2 + 11x + 24$

Therefore, the equation $x^2 + 11x + 24 = 0$ can be factored to $(\mathbf{x + 3})(\mathbf{x + 8}) = \mathbf{0}$.

Example 1.2-9

Solve the quadratic equation $9 = -x^2 - 6x$.

Solution:

First, write the equation in standard form, i.e., $x^2 + 6x + 9 = 0$.

Next, let: $\boxed{a = 1}$, $\boxed{b = 6}$, and $\boxed{c = 9}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}}$; $\boxed{x = \frac{-6 \pm \sqrt{36 - 36}}{2}}$; $\boxed{x = \frac{-6 \pm \sqrt{0}}{2}}$; $\boxed{x = \frac{-6 \pm 0}{2}}$

; $\boxed{x = -\frac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{x = -\frac{3}{1}}$; $\boxed{x = -3}$

In this case the equation has one repeated solution, i.e., $\boxed{\mathbf{x = -3}}$ and $\boxed{\mathbf{x = -3}}$.

Thus, the solution set is $\{\mathbf{-3, -3}\}$.

Check No. 1: $Let\ x = -3\ in \quad x^2 + 6x + 9 = 0$; $(-3)^2 + 6 \times -3 + 9 \overset{?}{=} 0$; $9 - 18 + 9 \overset{?}{=} 0$; $18 - 18 \overset{?}{=} 0$; $0 = 0$

Check No. 2: $x^2 + 6x + 9 \overset{?}{=} (x+3)(x+3)$; $x^2 + 6x + 9 \overset{?}{=} (x \cdot x) + (3 \cdot x) + (3 \cdot x) + (3 \cdot 3)$

; $x^2 + 6x + 9 \overset{?}{=} x^2 + 3x + 3x + 9$; $x^2 + 6x + 9 \overset{?}{=} x^2 + (3+3)x + 9$;

$x^2 + 6x + 9 = x^2 + 6x + 9$

Therefore, the equation $x^2 + 6x + 9 = 0$ can be factored to $(\mathbf{x+3})(\mathbf{x+3}) = \mathbf{0}$.

Example 1.2-10

Solve the quadratic equation $w^2 + 1 = -5w$.

Solution:

First, write the equation in standard form, i.e., $w^2 + 5w + 1 = 0$.

Next, let: $\boxed{a = 1}$, $\boxed{b = 5}$, and $\boxed{c = 1}$. Then,

Given: $\boxed{w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{w = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 1}}{2 \times 1}}$; $\boxed{w = \frac{-5 \pm \sqrt{25 - 4}}{2}}$; $\boxed{w = \frac{-5 \pm \sqrt{21}}{2}}$

; $\boxed{w = \frac{-5 \pm 4.58}{2}}$ Therefore:

I. $\boxed{w = \frac{-5 + 4.58}{2}}$; $\boxed{w = -\frac{0.42}{2}}$; $\boxed{\mathbf{w = -0.21}}$ II. $\boxed{w = \frac{-5 - 4.58}{2}}$; $\boxed{x = -\frac{9.58}{2}}$; $\boxed{\mathbf{w = -4.79}}$

and the solution set is $\{\mathbf{-0.21, -4.79}\}$.

Check No. 1: I. $Let\ w = -0.21\ in \quad w^2 + 1 = -5w$; $(-0.21)^2 + 1 \overset{?}{=} -5 \times -0.21$; $0.05 + 1 \overset{?}{=} 1.05$; $1.05 = 1.05$

II. $Let\ w = -4.79\ in \quad w^2 + 1 = -5w$; $(-4.79)^2 + 1 \overset{?}{=} -5 \times -4.79$; $22.9 + 1 \overset{?}{=} 23.9$; $23.9 = 23.9$

Check No. 2: $w^2 + 5w + 1 \overset{?}{=} (w + 0.21)(w + 4.79)$; $w^2 + 5w + 1 \overset{?}{=} (w \cdot w) + (4.79 \cdot w) + (0.21 \cdot w) + (0.21 \cdot 4.79)$

$$; \; w^2+5w+1 \stackrel{?}{=} w^2+4.79w+0.21w+1 \; ; \; w^2+5w+1 \stackrel{?}{=} w^2+(4.79+0.21)w+1$$

$$; \; w^2+5w+1 = w^2+5w+1$$

Therefore, the equation $w^2+5w+1=0$ can be factored to $\mathbf{(w+0.21)(w+4.79)=0}$.

Note that when $c=0$ the quadratic equation $ax^2+bx+c=0$ reduces to $ax^2+bx=0$. For cases where $a=1$, we can solve equations of the form $x^2+bx=0$ using the quadratic formula in the following way:

Example 1.2-11

Solve the quadratic equation $x^2+5x=0$.

Solution:

The equation is already in standard form.

Let: $\boxed{a=1}$, $\boxed{b=5}$, and $\boxed{c=0}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-5\pm\sqrt{5^2-4\times1\times0}}{2\times1}}$; $\boxed{x=\frac{-5\pm\sqrt{25-0}}{2}}$; $\boxed{x=\frac{-5\pm\sqrt{25}}{2}}$; $\boxed{x=\frac{-5\pm\sqrt{5^2}}{2}}$

; $\boxed{x=\frac{-5\pm5}{2}}$ Therefore:

I. $\boxed{x=\frac{-5+5}{2}}$; $\boxed{x=\frac{0}{2}}$; $\boxed{\mathbf{x=0}}$ II. $\boxed{x=\frac{-5-5}{2}}$; $\boxed{x=-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}}$; $\boxed{x=-\frac{5}{1}}$; $\boxed{x=-5}$; $\boxed{\mathbf{x=-5}}$

and the solution set is $\{\mathbf{0},\mathbf{-5}\}$.

Check No. 1: I. *Let* $x=0$ *in* $x^2+5x=0$; $0^2+5\cdot0 \stackrel{?}{=} 0$; $0+0 \stackrel{?}{=} 0$; $0=0$

II. *Let* $x=-5$ *in* $x^2+5x=0$; $(-5)^2+5\cdot-5 \stackrel{?}{=} 0$; $25-25 \stackrel{?}{=} 0$; $0=0$

Check No. 2: $x^2+5x \stackrel{?}{=} (x+0)(x+5)$; $x^2+5x \stackrel{?}{=} (x\cdot x)+(5\cdot x)+(0\cdot x)+(0\cdot5)$; $x^2+5x \stackrel{?}{=} x^2+5x+0+0$

; $x^2+5x=x^2+5x$

Therefore, the equation $x^2+5x=0$ can be factored to $(x+0)(x+5)=0$ which is the same as $\mathbf{x(x+5)=0}$.

Example 1.2-12

Solve the quadratic equation $x^2=9x$.

Solution:

First, write the equation in standard form, i.e., $x^2-9x=0$.

Next, let: $\boxed{a=1}$, $\boxed{b=-9}$, and $\boxed{c=0}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-(-9)\pm\sqrt{(-9)^2-4\times1\times0}}{2\times1}}$; $\boxed{x=\frac{9\pm\sqrt{81-0}}{2}}$; $\boxed{x=\frac{9\pm\sqrt{81}}{2}}$

; $\boxed{x = \dfrac{9 \pm \sqrt{9^2}}{2}}$; $\boxed{x = \dfrac{9 \pm 9}{2}}$ Therefore:

I. $\boxed{x = \dfrac{9-9}{2}}$; $\boxed{x = \dfrac{0}{2}}$; $\boxed{\mathbf{x = 0}}$

II. $\boxed{x = \dfrac{9+9}{2}}$; $\boxed{x = \dfrac{\overset{9}{\cancel{18}}}{\cancel{2}}}$; $\boxed{x = \dfrac{9}{1}}$; $\boxed{\mathbf{x = 9}}$

and the solution set is $\{\mathbf{0}, \mathbf{9}\}$.

Check No. 1: I. *Let* $x = 0$ *in* $x^2 = 9x$; $0^2 \stackrel{?}{=} 9 \cdot 0$; $0 = 0$

II. *Let* $x = 9$ *in* $x^2 = 9x$; $9^2 \stackrel{?}{=} 9 \cdot 9$; $81 = 81$

Check No. 2: $x^2 - 9x \stackrel{?}{=} (x+0)(x-9)$; $x^2 - 9x \stackrel{?}{=} (x \cdot x) + (-9 \cdot x) + (0 \cdot x) + (0 \cdot -9)$; $x^2 - 9x \stackrel{?}{=} x^2 - 9x + 0 + 0$; $x^2 - 9x = x^2 - 9x$

Therefore, the equation $x^2 - 9x = 0$ can be factored to $(x+0)(x-9) = 0$ which is the same as $\boldsymbol{x(x-9) = 0}$.

Practice Problems - Solving Quadratic Equations of the Form $ax^2 + bx + c$, where $a = 1$, Using the Quadratic Formula

Section 1.2 Case I Practice Problems - Use the quadratic formula to solve the following quadratic equations.

1. $x^2 = -5x - 6$
2. $y^2 - 40y = -300$
3. $-x = -x^2 + 20$
4. $x^2 + 3x + 4 = 0$
5. $x^2 - 80 - 2x = 0$
6. $x^2 + 4x + 4 = 0$
7. $-6 = -w^2 + w$
8. $4x = x^2$
9. $z^2 - 37z - 120 = 0$
10. $x^2 - 20 = -8x$

Case II Solving Quadratic Equations of the Form $ax^2+bx+c=0$, where $a \rangle 1$, Using the Quadratic Equation

Trinomial equations of the form $ax^2+bx+c=0$, where $a \rangle 1$, are solved using the following steps:

Step 1 Write the equation in standard form.

Step 2 Identify the coefficients a, b, and c.

Step 3 Substitute the values for a, b, and c into the quadratic equation $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Simplify the equation.

Step 4 Solve for the values of x. Check the answers by either substituting the x values into the original equation or by multiplying the factored product using the FOIL method.

Step 5 Write the quadratic equation in its factored form.

Examples with Steps

The following examples show the steps as to how second degree trinomial equations are solved using the quadratic formula:

Example 1.2-13

Solve the quadratic equation $2x^2+5x=-3$.

Solution:

Step 1 $\boxed{2x^2+5x=-3}$; $\boxed{2x^2+5x+3=0}$

Step 2 Let: $\boxed{a=2}$, $\boxed{b=5}$, and $\boxed{c=3}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-5\pm\sqrt{5^2-4\times 2\times 3}}{2\times 2}}$; $\boxed{x=\frac{-5\pm\sqrt{25-24}}{4}}$

; $\boxed{x=\frac{-5\pm\sqrt{1}}{4}}$; $\boxed{x=\frac{-5\pm 1}{4}}$

Step 4 Separate $x=\frac{-5\pm 1}{4}$ into two equations:

I. $\boxed{x=\frac{-5+1}{4}}$; $\boxed{x=-\frac{\cancel{4}}{\cancel{4}}}$; $\boxed{x=-\frac{1}{1}}$; $\boxed{\boldsymbol{x=-1}}$ II. $\boxed{x=\frac{-5-1}{4}}$; $\boxed{x=-\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}}$; $\boxed{\boldsymbol{x=-\frac{3}{2}}}$

Thus, the solution set is $\left\{\mathbf{-1,-\frac{3}{2}}\right\}$.

Check No. 1: I. *Let* $x=-1$ *in* $2x^2+5x=-3$; $2(-1)^2+(5\times -1)\stackrel{?}{=}-3$; $2-5\stackrel{?}{=}-3$; $-3=-3$

II. *Let* $x=-\frac{3}{2}$ *in* $2x^2+5x=-3$; $2\left(-\frac{3}{2}\right)^2+\left(5\times -\frac{3}{2}\right)\stackrel{?}{=}-3$; $2\times\frac{9}{4}-\frac{15}{2}\stackrel{?}{=}-3$

; $\frac{18}{4}-\frac{15}{2}\stackrel{?}{=}-3$; $\frac{(2\times 18)-(4\times 15)}{4\times 2}\stackrel{?}{=}-3$; $\frac{36-60}{8}\stackrel{?}{=}-3$; $-\frac{24}{8}\stackrel{?}{=}-3$; $-3=-3$

Check No. 2: $2x^2+5x+3\overset{?}{=}(x+1)(2x+3)$; $2x^2+5x+3\overset{?}{=}(2x\cdot x)+(3\cdot x)+(1\cdot 2x)+(1\cdot 3)$

; $2x^2+5x+3\overset{?}{=}2x^2+3x+2x+3$; $2x^2+5x+3\overset{?}{=}2x^2+(3+2)x+3$

; $2x^2+5x+3=2x^2+5x+3$

Step 5 Therefore, the equation $2x^2+5x+3=0$ can be factored to $(x+1)\left(x+\frac{3}{2}\right)=0$

which is the same as $\mathbf{(x+1)(2x+3)=0}$

Example 1.2-14

Solve the quadratic equation $15x^2=-7x+2$.

Solution:

Step 1 $\boxed{15x^2=-7x+2}$; $\boxed{15x^2+7x=-7x+7x+2}$; $\boxed{15x^2+7x=0+2}$; $\boxed{15x^2+7x=2}$

; $\boxed{15x^2+7x-2=2-2}$; $\boxed{15x^2+7x-2=0}$

Step 2 Let: $\boxed{a=15}$, $\boxed{b=7}$, and $\boxed{c=-2}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-7\pm\sqrt{7^2-4\times15\times-2}}{2\times15}}$; $\boxed{x=\frac{-7\pm\sqrt{49+120}}{30}}$

; $\boxed{x=\frac{-7\pm\sqrt{169}}{30}}$; $\boxed{x=\frac{-7\pm13}{30}}$

Step 4 Separate $x=\frac{-7\pm13}{30}$ into two equations:

I. $\boxed{x=\frac{-7+13}{30}}$; $\boxed{x=\frac{\cancel{6}}{\underset{5}{\cancel{30}}}}$; $\boxed{\mathbf{x=\frac{1}{5}}}$ II. $\boxed{x=\frac{-7-13}{30}}$; $\boxed{x=-\frac{\overset{2}{\cancel{20}}}{\underset{3}{\cancel{30}}}}$; $\boxed{\mathbf{x=-\frac{2}{3}}}$

Thus, the solution set is $\left\{-\frac{\mathbf{2}}{\mathbf{3}},\frac{\mathbf{1}}{\mathbf{5}}\right\}$.

Check No. 1: I. *Let* $x=\frac{1}{5}$ *in* $15x^2=-7x+2$; $15\left(\frac{1}{5}\right)^2\overset{?}{=}\left(-7\times\frac{1}{5}\right)+2$; $15\times\frac{1}{25}\overset{?}{=}-\frac{7}{5}+2$

; $\frac{\overset{3}{\cancel{15}}}{\underset{5}{\cancel{25}}}\overset{?}{=}-\frac{7}{5}+\frac{2}{1}$; $\frac{3}{5}\overset{?}{=}\frac{(-7\times1)+(2\times5)}{5\times1}$; $\frac{3}{5}\overset{?}{=}\frac{-7+10}{5}$; $\frac{3}{5}=\frac{3}{5}$

II. *Let* $x=-\frac{2}{3}$ *in* $15x^2=-7x+2$; $15\left(-\frac{2}{3}\right)^2\overset{?}{=}\left(-7\times-\frac{2}{3}\right)+2$; $15\times\frac{4}{9}\overset{?}{=}\frac{14}{3}+2$

; $\frac{\overset{20}{\cancel{60}}}{\underset{3}{\cancel{9}}}\overset{?}{=}\frac{14}{3}+\frac{2}{1}$; $\frac{20}{3}\overset{?}{=}\frac{(14\times1)+(2\times3)}{3\times1}$; $\frac{20}{3}\overset{?}{=}\frac{14+6}{3}$; $\frac{20}{3}=\frac{20}{3}$

Check No. 2: $15x^2+7x-2\overset{?}{=}(5x-1)(3x+2)$; $15x^2+7x-2\overset{?}{=}(5x\cdot3x)+(2\cdot5x)+(-1\cdot3x)+(-1\cdot2)$

; $15x^2+7x-2\overset{?}{=}15x^2+10x-3x-2$; $15x^2+7x-2\overset{?}{=}15x^2+(10-3)x-2$

; $15x^2+7x-2=15x^2+7x-2$

Step 5 Therefore, the equation $15x^2+7x-2=0$ can be factored to $\left(x-\frac{1}{5}\right)\left(x+\frac{2}{3}\right)=0$ which is the same as $\mathbf{(5x-1)(3x+2)=0}$

Example 1.2-15

Solve the quadratic equation $4x^2+4xy=3y^2$. Let x be the variable.

Solution:

Step 1 $\boxed{4x^2+4xy=3y^2}$; $\boxed{4x^2+4xy-3y^2=3y^2-3y^2}$; $\boxed{4x^2+4yx-3y^2=0}$

Step 2 Let: $\boxed{a=4}$, $\boxed{b=4y}$, and $\boxed{c=-3y^2}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-(4y)\pm\sqrt{(4y)^2-4\times4\times-3y^2}}{2\times4}}$

; $\boxed{x=\frac{-4y\pm\sqrt{16y^2+48y^2}}{8}}$; $\boxed{x=\frac{-4y\pm\sqrt{64y^2}}{8}}$; $\boxed{x=\frac{-4y\pm8y}{8}}$

Step 4 Separate $x=\frac{-4y\pm8y}{8}$ into two equations:

I. $\boxed{x=\frac{-4y+8y}{8}}$; $\boxed{x=\frac{\cancel{4}y}{\underset{2}{\cancel{8}}}}$; $\boxed{\mathbf{x=\frac{y}{2}}}$ II. $\boxed{x=\frac{-4y-8y}{8}}$; $\boxed{x=-\frac{\overset{3}{\cancel{12}}y}{\underset{2}{\cancel{8}}}}$; $\boxed{\mathbf{x=-\frac{3y}{2}}}$

Thus, the solution set is $\left\{-\frac{\mathbf{3y}}{\mathbf{2}},\frac{\mathbf{y}}{\mathbf{2}}\right\}$.

Check No. 1: I. *Let* $x=\frac{y}{2}$ *in* $4x^2+4xy=3y^2$; $4\left(\frac{y}{2}\right)^2+\left(4\times\frac{y}{2}\times y\right)\overset{?}{=}3y^2$; $\cancel{4}\times\frac{y^2}{\cancel{4}}+\frac{\cancel{4}y^2}{\cancel{2}}\overset{?}{=}3y^2$

; $y^2+2y^2\overset{?}{=}3y^2$; $3y^2=3y^2$

II. *Let* $x=-\frac{3y}{2}$ *in* $4x^2+4xy=3y^2$; $4\left(-\frac{3y}{2}\right)^2+\left(4\times-\frac{3y}{2}\times y\right)\overset{?}{=}3y^2$

; $\cancel{4}\times\frac{9y^2}{\cancel{4}}-\frac{\overset{6}{\cancel{12}}y^2}{\cancel{2}}\overset{?}{=}3y^2$; $9y^2-6y^2\overset{?}{=}3y^2$; $3y^2=3y^2$

Check No. 2: $4x^2+4yx-3y^2\overset{?}{=}(2x-y)(2x+3y)$; $4x^2+4yx-3y^2\overset{?}{=}(2x\cdot2x)+(2x\cdot3y)+(2x\cdot-y)$ $+(-y\cdot3y)$; $4x^2+4yx-3y^2\overset{?}{=}4x^2+6xy-2xy-3y^2$; $4x^2+4yx-3y^2\overset{?}{=}4x^2$ $+(6-2)xy-3y^2$; $4x^2+4yx-3y^2=4x^2+4xy-3y^2$

Step 5 Therefore, the equation $4x^2+4yx-3y^2=0$ can be factored to $\left(x-\frac{y}{2}\right)\left(x+\frac{3y}{2}\right)=0$ which is the same as $\mathbf{(2x-y)(2x+3y)=0}$

Example 1.2-16

Solve the quadratic equation $2x^2+15=13x$.

Solution:

Step 1 $\boxed{2x^2+15=13x}$; $\boxed{2x^2-13x+15=13x-13x}$; $\boxed{2x^2-13x+15=0}$

Step 2 Let: $\boxed{a=2}$, $\boxed{b=-13}$, and $\boxed{c=15}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-(-13)\pm\sqrt{(-13)^2-(4\times2\times15)}}{2\times2}}$

; $\boxed{x=\frac{13\pm\sqrt{169-120}}{4}}$; $\boxed{x=\frac{13\pm\sqrt{49}}{4}}$; $\boxed{x=\frac{13\pm7}{4}}$

Step 4 Separate $x=\frac{13\pm7}{4}$ into two equations:

I. $\boxed{x=\frac{13+7}{4}}$; $\boxed{x=\frac{\overset{5}{\cancel{20}}}{\cancel{4}}}$; $\boxed{x=\frac{5}{1}}$; $\boxed{\boldsymbol{x=5}}$ II. $\boxed{x=\frac{13-7}{4}}$; $\boxed{x=\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}}$; $\boxed{\boldsymbol{x=\frac{3}{2}}}$

Thus, the solution set is $\left\{\boldsymbol{\frac{3}{2}, 5}\right\}$.

Check No. 1: I. $Let\ x=5\ in\quad 2x^2+15=13x$; $2(5)^2+15\overset{?}{=}13\times5$; $2\times25+15\overset{?}{=}65$; $50+15\overset{?}{=}65$

; $65=65$

II. $Let\ x=\frac{3}{2}\ in\quad 2x^2+15=13x$; $2\left(\frac{3}{2}\right)^2+15\overset{?}{=}13\times\frac{3}{2}$; $\cancel{2}\times\frac{9}{\cancel{4}}+15\overset{?}{=}\frac{39}{2}$; $\frac{9}{2}+15\overset{?}{=}\frac{39}{2}$

; $\frac{9}{2}+\frac{15}{1}\overset{?}{=}\frac{39}{2}$; $\frac{(1\times9)+(2\times15)}{1\times2}\overset{?}{=}\frac{39}{2}$; $\frac{9+30}{2}\overset{?}{=}\frac{39}{2}$; $\frac{39}{2}=\frac{39}{2}$

Check No. 2: $2x^2-13x+15\overset{?}{=}(x-5)(2x-3)$; $2x^2-13x+15\overset{?}{=}(x\cdot2x)+(x\cdot-3)+(-5\cdot2x)+(-5\cdot-3)$

; $2x^2-13x+15\overset{?}{=}2x^2-3x-10x+15$; $2x^2-13x+15\overset{?}{=}2x^2+(-3-10)x+15$

; $2x^2-13x+15=2x^2-13x+15$

Step 5 Thus, the equation $2x^2-13x+15=0$ can be factored to $(x-5)\left(x-\frac{3}{2}\right)=0$

which is the same as $\boldsymbol{(x-5)(2x-3)=0}$

Example 1.2-17

Solve the quadratic equation $4x^2-15x-4=0$.

Solution:

Step 1 $\boxed{\textit{Not Applicable}}$

Step 2 Let: $\boxed{a=4}$, $\boxed{b=-15}$, and $\boxed{c=-4}$. Then,

Step 3 Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-(-15)\pm\sqrt{(-15)^2-(4\times4\times-4)}}{2\times4}}$

; $\boxed{x=\frac{15\pm\sqrt{225+64}}{8}}$; $\boxed{x=\frac{15\pm\sqrt{289}}{8}}$; $\boxed{x=\frac{15\pm17}{8}}$

Step 4 Separate $x=\frac{15\pm 17}{8}$ into two equations:

I. $\boxed{x=\frac{15+17}{8}}$; $\boxed{x=\frac{\cancel{32}^{\,4}}{\cancel{8}}}$; $\boxed{\mathbf{x=4}}$ II. $\boxed{x=\frac{15-17}{8}}$; $\boxed{x=-\frac{\cancel{2}}{\cancel{8}_{\,4}}}$; $\boxed{\mathbf{x=-\frac{1}{4}}}$

Thus, the solution set is $\left\{-\frac{\mathbf{1}}{\mathbf{4}}, \mathbf{4}\right\}$.

Check No. 1: I. *Let* $x=4$ *in* $4x^2-15x-4=0$; $\left(4\times 4^2\right)-(15\times 4)-4\stackrel{?}{=}0$; $(4\times 16)-60-4\stackrel{?}{=}0$

; $64-60-4\stackrel{?}{=}0$; $64-64\stackrel{?}{=}0$; $0=0$

II. *Let* $x=-\frac{1}{4}$ *in* $4x^2-15x-4=0$; $4\times\left(-\frac{1}{4}\right)^2-\left(15\times -\frac{1}{4}\right)-4\stackrel{?}{=}0$; $\cancel{4}\times\frac{1}{\cancel{16}}+\frac{15}{4}-4\stackrel{?}{=}0$

; $\frac{1}{4}+\frac{15}{4}-4\stackrel{?}{=}0$; $\frac{1+15}{4}-4\stackrel{?}{=}0$; $\frac{\cancel{16}}{\cancel{4}}-4\stackrel{?}{=}0$; $\frac{4}{1}-4\stackrel{?}{=}0$; $4-4\stackrel{?}{=}0$; $0=0$

Check No. 2: $4x^2-15x-4\stackrel{?}{=}(x-4)\left(x+\frac{1}{4}\right)=0$; $4x^2-15x-4\stackrel{?}{=}(x\cdot x)+\left(\frac{1}{4}\cdot x\right)+(-4\cdot x)+\left(-\cancel{4}\cdot\frac{1}{\cancel{4}}\right)$

; $4x^2-15x-4\stackrel{?}{=}x^2+\frac{x}{4}-4x-1$; $4x^2-15x-4\stackrel{?}{=}x^2+\left(\frac{x}{4}-\frac{4x}{1}\right)-1$

; $4x^2-15x-4\stackrel{?}{=}x^2+\left(\frac{(1\cdot x)-(4x\cdot 4)}{1\cdot 4}\right)-1$; $4x^2-15x-4\stackrel{?}{=}x^2+\left(\frac{x-16x}{4}\right)-1$

; $4x^2-15x-4\stackrel{?}{=}x^2-\frac{15}{4}x-1$; $4x^2-15x-4\stackrel{?}{=}4\cdot\left(x^2-\frac{15}{4}x-1\right)$

; $4x^2-15x-4=4x^2-15x-4$

Step 5 Thus, the equation $4x^2-15x-4=0$ can be factored to $(x-4)\left(x+\frac{1}{4}\right)=0$ which is the same as $\mathbf{(x-4)(4x+1)=0}$.

Additional Examples - Solving Quadratic Equations of the Form $ax^2+bx+c=0$, where $a\rangle 1$, Using the Quadratic Formula

The following examples further illustrate how to solve quadratic equations:

Example 1.2-18

Solve the quadratic equation $3x^2+7x-6=0$.

Solution:

The equation is already in standard form. Let: $\boxed{a=3}$, $\boxed{b=7}$, and $\boxed{c=-6}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-7\pm\sqrt{7^2-4\times 3\times -6}}{2\times 3}}$; $\boxed{x=\frac{-7\pm\sqrt{49+72}}{6}}$; $\boxed{x=\frac{-7\pm\sqrt{121}}{6}}$

; $\boxed{x=\frac{-7\pm\sqrt{11^2}}{6}}$; $\boxed{x=\frac{-7\pm 11}{6}}$ Therefore:

I. $\boxed{x=\frac{-7+11}{6}}$; $\boxed{x=\frac{\cancel{4}^{\,2}}{\cancel{6}_{\,3}}}$; $\boxed{\mathbf{x=\frac{2}{3}}}$ II. $\boxed{x=\frac{-7-11}{6}}$; $\boxed{x=-\frac{\cancel{18}^{\,3}}{\cancel{6}}}$; $\boxed{x=-\frac{3}{1}}$; $\boxed{\mathbf{x=-3}}$

Thus, the solution set is $\left\{\mathbf{-3}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$.

Check No. 1: I. *Let* $x=-3$ *in* $3x^2+7x-6=0$; $3\cdot(-3)^2+7\cdot(-3)-6\overset{?}{=}0$; $3\cdot 9-21-6\overset{?}{=}0$; $27-21-6\overset{?}{=}0$; $27-27\overset{?}{=}0$; $0=0$

II. *Let* $x=\frac{2}{3}$ *in* $3x^2+7x-6=0$; $3\cdot\left(\frac{2}{3}\right)^2+7\cdot\left(\frac{2}{3}\right)-6\overset{?}{=}0$; $3\cdot\frac{4}{9}+\frac{14}{3}-6\overset{?}{=}0$; $\frac{\overset{4}{\cancel{12}}}{\underset{3}{\cancel{9}}}+\frac{14}{3}-6\overset{?}{=}0$; $\frac{4}{3}+\frac{14}{3}-6\overset{?}{=}0$; $\frac{4+14}{3}-6\overset{?}{=}0$; $\frac{\overset{6}{\cancel{18}}}{\cancel{3}}-6\overset{?}{=}0$; $\frac{6}{1}-6\overset{?}{=}0$; $6-6\overset{?}{=}0$; $0=0$

Check No. 2: $3x^2+7x-6\overset{?}{=}(x+3)(3x-2)$; $3x^2+7x-6\overset{?}{=}(x\cdot 3x)+(-2\cdot x)+(3\cdot 3x)+(3\cdot -2)$; $3x^2+7x-6\overset{?}{=}3x^2-2x+9x-6$; $3x^2+7x-6\overset{?}{=}3x^2+(-2+9)x-6$; $3x^2+7x-6=3x^2+7x-6$

Therefore, the equation $3x^2+7x-6=0$ can be factored to $(x+3)\left(x-\frac{2}{3}\right)=0$ which is the same as $(x+3)\left(\frac{x}{1}-\frac{2}{3}\right)=0$; $(x+3)\left(\frac{(3\cdot x)-(1\cdot 2)}{1\cdot 3}\right)=0$; $(x+3)\left(\frac{3x-2}{3}\right)=0$; $\left(\frac{x+3}{1}\right)\left(\frac{3x-2}{3}\right)=0$; $\frac{(x+3)\cdot(3x-2)}{1\cdot 3}=0$; $\frac{(x+3)\cdot(3x-2)}{1\cdot 3}=\frac{0}{1}$; $[(x+3)\cdot(3x-2)]\cdot 1=0\cdot 3$; $\mathbf{(x+3)(3x-2)=0}$

Example 1.2-19

Solve the quadratic equation $6x^2=-7x-2$.

Solution:

First, write the equation in standard form, i.e., $6x^2+7x+2=0$

Next, let: $\boxed{a=6}$, $\boxed{b=7}$, and $\boxed{c=2}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-7\pm\sqrt{7^2-4\times 6\times 2}}{2\times 6}}$; $\boxed{x=\frac{-7\pm\sqrt{49-48}}{12}}$; $\boxed{x=\frac{-7\pm\sqrt{1}}{12}}$; $\boxed{x=\frac{-7\pm 1}{12}}$

Therefore: I. $\boxed{x=\frac{-7+1}{12}}$; $\boxed{x=-\frac{\cancel{6}}{\underset{2}{\cancel{12}}}}$; $\boxed{\mathbf{x=-\frac{1}{2}}}$ II. $\boxed{x=\frac{-7-1}{12}}$; $\boxed{x=-\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}}$; $\boxed{\mathbf{x=-\frac{2}{3}}}$

Thus, the solution set is $\left\{-\frac{\mathbf{1}}{\mathbf{2}}, -\frac{\mathbf{2}}{\mathbf{3}}\right\}$.

Check No. 1: I. *Let* $x=-\frac{1}{2}$ *in* $6x^2=-7x-2$; $6\left(-\frac{1}{2}\right)^2\overset{?}{=}\left(-7\times-\frac{1}{2}\right)-2$; $6\cdot\frac{1}{4}\overset{?}{=}\frac{7}{2}-2$; $\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}\overset{?}{=}\frac{7}{2}-\frac{2}{1}$; $\frac{3}{2}\overset{?}{=}\frac{(7\cdot 1)-(2\cdot 2)}{2\cdot 1}$; $\frac{3}{2}\overset{?}{=}\frac{7-4}{2}$; $\frac{3}{2}=\frac{3}{2}$

II. $Let\ x = -\frac{2}{3}\ in\quad 6x^2 = -7x - 2$; $6\left(-\frac{2}{3}\right)^2 \overset{?}{=} \left(-7 \times -\frac{2}{3}\right) - 2$; $6 \cdot \frac{4}{9} \overset{?}{=} \frac{14}{3} - 2$; $\frac{\overset{8}{\cancel{24}}}{\underset{3}{\cancel{9}}} \overset{?}{=} \frac{14}{3} - \frac{2}{1}$

; $\frac{8}{3} \overset{?}{=} \frac{(14 \cdot 1) - (2 \cdot 3)}{3 \cdot 1}$; $\frac{8}{3} \overset{?}{=} \frac{14 - 6}{3}$; $\frac{8}{3} = \frac{8}{3}$

Check No. 2: $6x^2 + 7x + 2 \overset{?}{=} (2x+1)(3x+2)$; $6x^2 + 7x + 2 \overset{?}{=} (2x \cdot 3x) + (2 \cdot 2x) + (1 \cdot 3x) + (1 \cdot 2)$

; $6x^2 + 7x + 2 \overset{?}{=} 6x^2 + 4x + 3x + 2$; $6x^2 + 7x + 2 \overset{?}{=} 6x^2 + (4+3)x + 2$

; $6x^2 + 7x + 2 = 6x^2 + 7x + 2$

Therefore, the equation $6x^2 + 7x + 2 = 0$ can be factored to $\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right) = 0$ which is the same as $\left(\frac{x}{1} + \frac{1}{2}\right)\left(\frac{x}{1} + \frac{2}{3}\right) = 0$; $\left(\frac{(2 \cdot x) + (1 \cdot 1)}{1 \cdot 2}\right)\left(\frac{(3 \cdot x) + (1 \cdot 2)}{1 \cdot 3}\right) = 0$; $\left(\frac{2x+1}{2}\right)\left(\frac{3x+2}{3}\right) = 0$; $\frac{(2x+1) \cdot (3x+2)}{2 \cdot 3} = 0$

; $\frac{(2x+1)(3x+2)}{6} = \frac{0}{1}$; $[(2x+1) \cdot (3x+2)] \cdot 1 = 0 \cdot 6$; $\mathbf{(2x+1)(3x+2) = 0}$

Example 1.2-20

Solve the quadratic equation $-16x + 5 = -3x^2$.

Solution:

First, write the equation in standard form, i.e., $3x^2 - 16x + 5 = 0$.

Next, let: $\boxed{a = 3}$, $\boxed{b = -16}$, and $\boxed{c = 5}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \times 3 \times 5}}{2 \times 3}}$; $\boxed{x = \frac{16 \pm \sqrt{256 - 60}}{6}}$; $\boxed{x = \frac{16 \pm \sqrt{196}}{6}}$

; $\boxed{x = \frac{16 \pm \sqrt{14^2}}{6}}$; $\boxed{x = \frac{16 \pm 14}{6}}$ Therefore:

I. $\boxed{x = \frac{16+14}{6}}$; $\boxed{x = \frac{\overset{5}{\cancel{30}}}{\cancel{6}}}$; $\boxed{x = \frac{5}{1}}$; $\boxed{\mathbf{x = 5}}$

II. $\boxed{x = \frac{16-14}{6}}$; $\boxed{x = \frac{\cancel{2}}{\underset{3}{\cancel{6}}}}$; $\boxed{\mathbf{x = \frac{1}{3}}}$

Thus, the solution set is $\left\{\mathbf{\frac{1}{3}, 5}\right\}$.

Check No. 1: I. $Let\ x = \frac{1}{3}\ in\quad -16x + 5 = -3x^2$; $-16 \cdot \frac{1}{3} + 5 \overset{?}{=} -3 \cdot \left(\frac{1}{3}\right)^2$; $-\frac{16}{3} + 5 \overset{?}{=} -3 \cdot \frac{1}{9}$

; $-\frac{16}{3} + \frac{5}{1} \overset{?}{=} -\frac{\cancel{3}}{\underset{3}{\cancel{9}}}$; $\frac{(-16 \cdot 1) + (5 \cdot 3)}{3 \cdot 1} \overset{?}{=} -\frac{1}{3}$; $\frac{-16 + 15}{3} \overset{?}{=} -\frac{1}{3}$; $-\frac{1}{3} = -\frac{1}{3}$

II. $Let\ x = 5\ in\quad -16x + 5 = -3x^2$; $-16 \cdot 5 + 5 \overset{?}{=} -3 \cdot 5^2$; $-80 + 5 \overset{?}{=} -3 \cdot 25$; $-75 = -75$

Check No. 2: $3x^2 - 16x + 5 \overset{?}{=} \left(x - \frac{1}{3}\right)(x - 5)$; $3x^2 - 16x + 5 \overset{?}{=} (x \cdot x) + (-5 \cdot x) + \left(-\frac{1}{3} \cdot x\right) + \left(-\frac{1}{3} \cdot -5\right)$

; $3x^2 - 16x + 5 \overset{?}{=} x^2 - 5x - \frac{1}{3}x + \frac{5}{3}$; $3x^2 - 16x + 5 \overset{?}{=} x^2 + \left(-5 - \frac{1}{3}\right)x + \frac{5}{3}$

$$; \ 3x^2 - 16x + 5 \stackrel{?}{=} x^2 + \left(-\frac{5}{1} - \frac{1}{3}\right)x + \frac{5}{3} \ ; \ 3x^2 - 16x + 5 \stackrel{?}{=} x^2 + \left(\frac{(-5 \cdot 3) - (1 \cdot 1)}{1 \cdot 3}\right)x + \frac{5}{3}$$

$$; \ 3x^2 - 16x + 5 \stackrel{?}{=} x^2 + \left(\frac{-15-1}{3}\right)x + \frac{5}{3} \ ; \ 3x^2 - 16x + 5 \stackrel{?}{=} x^2 - \frac{16}{3}x + \frac{5}{3}$$

$$; \ 3x^2 - 16x + 5 \stackrel{?}{=} 3 \cdot \left(x^2 - \frac{16}{3}x + \frac{5}{3}\right) \ ; \ 3x^2 - 16x + 5 = 3x^2 - 16x + 5$$

Therefore, the equation $3x^2 - 16x + 5 = 0$ can be factored to $\left(x - \frac{1}{3}\right)(x-5) = 0$ which is the same as $\mathbf{(3x-1)(x-5)=0}$.

Example 1.2-21

Solve the quadratic equation $4x^2 + 9x = -6$.

Solution:

First, write the equation in standard form, i.e., $4x^2 + 9x + 6 = 0$.

Next, let: $\boxed{a=4}$, $\boxed{b=9}$, and $\boxed{c=6}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-9 \pm \sqrt{9^2 - 4 \times 4 \times 6}}{2 \times 4}}$; $\boxed{x = \frac{-9 \pm \sqrt{81-96}}{8}}$; $\boxed{x = \frac{-9 \pm \sqrt{-15}}{8}}$

Since the number under the radical is negative, therefore the quadratic equation does not have any real solutions. We state that **the equation is not factorable**.

Example 1.2-22

Solve the quadratic equation $3y^2 - 2y = 2$.

Solution:

First, write the equation in standard form, i.e., $3y^2 - 2y - 2 = 0$.

Next, let: $\boxed{a=3}$, $\boxed{b=-2}$, and $\boxed{c=-2}$. Then,

Given: $\boxed{y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -2}}{2 \times 3}}$; $\boxed{y = \frac{2 \pm \sqrt{4+24}}{6}}$; $\boxed{y = \frac{2 \pm \sqrt{28}}{6}}$

; $\boxed{y = \frac{2 \pm 5.3}{6}}$ Therefore:

I. $\boxed{y = \frac{2+5.3}{6}}$; $\boxed{y = \frac{7.3}{6}}$; $\boxed{\mathbf{y = 1.22}}$ II. $\boxed{y = \frac{2-5.3}{6}}$; $\boxed{y = -\frac{3.3}{6}}$; $\boxed{\mathbf{y = -0.55}}$

Thus, the solution set is $\{\mathbf{-0.55, 1.22}\}$.

Check No. 1: I. *Let* $y = 1.22$ *in* $3y^2 - 2y = 2$; $3 \cdot (1.22)^2 - 2 \cdot 1.22 \stackrel{?}{=} 2$; $3 \cdot 1.48 - 2.44 \stackrel{?}{=} 2$

; $4.44 - 2.44 \stackrel{?}{=} 2$; $2 = 2$

II. *Let* $y = -0.55$ *in* $3y^2 - 2y = 2$; $3 \cdot (-0.55)^2 - 2 \cdot (-0.55) \stackrel{?}{=} 2$; $3 \cdot 0.3 + 1.1 \stackrel{?}{=} 2$

; $0.9 + 1.1 \stackrel{?}{=} 2$; $2 = 2$

Check No. 2: $3y^2-2y-2\overset{?}{=}(y+0.55)(y-1.22)$; $3y^2-2y-2\overset{?}{=}(y\cdot y)+(-1.22\cdot y)+(0.55\cdot y)+(0.55\cdot -1.22)$

; $3y^2-2y-2\overset{?}{=}y^2-1.22y+0.55y-0.67$; $3y^2-2y-2\overset{?}{=}y^2+(-1.22+0.55)y-0.67$

; $3y^2-2y-2\overset{?}{=}y^2-0.67y-0.67$; $3y^2-2y-2\overset{?}{=}3\cdot\left(y^2-0.67y-0.67\right)$

; $3y^2-2y-2=3y^2-2y-2$

Therefore, the equation $3y^2-2y-2=0$ can be factored to $\mathbf{(y+0.55)(y-1.22)=0}$.

Note that when $c=0$ the quadratic equation $ax^2+bx+c=0$ reduces to $ax^2+bx=0$. For cases where $a\rangle 1$, we can solve equations of the form $ax^2+bx=0$ using the quadratic formula in the following way:

Example 1.2-23

Solve the quadratic equation $2x^2+5x=0$.

Solution:

First write the equation in standard form, i.e., $2x^2+5x+0=0$.

Next, let: $\boxed{a=2}$, $\boxed{b=5}$, and $\boxed{c=0}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-5\pm\sqrt{5^2-4\times2\times0}}{2\times2}}$; $\boxed{x=\frac{-5\pm\sqrt{25-0}}{4}}$; $\boxed{x=\frac{-5\pm\sqrt{25}}{4}}$

; $\boxed{x=\frac{-5\pm\sqrt{5^2}}{4}}$; $\boxed{x=\frac{-5\pm5}{4}}$ Therefore:

I. $\boxed{x=\frac{-5+5}{4}}$; $\boxed{x=\frac{0}{4}}$; $\boxed{\mathbf{x=0}}$

II. $\boxed{x=\frac{-5-5}{4}}$; $\boxed{x=-\frac{\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}}}$; $\boxed{x=-\frac{5}{2}}$; $\boxed{\mathbf{x=-2.5}}$

Thus, the solution set is $\{\mathbf{0,-2.5}\}$.

Check No. 1: I. *Let* $x=0$ *in* $2x^2+5x=0$; $2\cdot0^2+5\cdot0\overset{?}{=}0$; $0+0\overset{?}{=}0$; $0=0$

II. *Let* $x=-2.5$ *in* $2x^2+5x=0$; $2\cdot(-2.5)^2+5\cdot-2.5\overset{?}{=}0$; $2\cdot6.25-12.5\overset{?}{=}0$; $12.5=12.5$

Check No. 2: $2x^2+5x\overset{?}{=}(x+0)(x+2.5)$; $2x^2+5x\overset{?}{=}(x\cdot x)+(2.5\cdot x)+(0\cdot x)+(0\cdot2.5)$

; $2x^2+5x\overset{?}{=}x^2+2.5x+0+0$; $2x^2+5x\overset{?}{=}x^2+2.5x$; $2x^2+5x\overset{?}{=}2\left(x^2+2.5x\right)$

; $2x^2+5x=2x^2+5x$

Therefore, the equation $2x^2+5x=0$ can be factored to $(x+0)(x+2.5)=0$ which is the same as $\mathbf{x(x+2.5)=0}$.

Example 1.2-24

Solve the quadratic equation $3x^2=2x$.

Solution:

First, write the equation in standard form, i.e., $3x^2-2x+0=0$.

Next, let: $\boxed{a=3}$, $\boxed{b=-2}$, and $\boxed{c=0}$. Then,

Given: $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$; $\boxed{x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times 0}}{2 \times 3}}$; $\boxed{x = \frac{2 \pm \sqrt{4-0}}{6}}$; $\boxed{x = \frac{2 \pm \sqrt{4}}{6}}$

; $\boxed{x = \frac{2 \pm \sqrt{2^2}}{6}}$; $\boxed{x = \frac{2 \pm 2}{6}}$ Therefore:

I. $\boxed{x = \frac{2-2}{6}}$; $\boxed{x = \frac{0}{6}}$; $\boxed{\mathbf{x = 0}}$ II. $\boxed{x = \frac{2+2}{6}}$; $\boxed{x = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}}}$; $\boxed{x = \frac{2}{3}}$; $\boxed{\mathbf{x = 0.67}}$

Thus, the solution set is $\{\mathbf{0, 0.67}\}$.

Check No. 1: I. $Let\ x = 0\ \ in \quad 3x^2 = 2x$; $3 \cdot 0^2 \overset{?}{=} 2 \cdot 0$; $0 = 0$

II. $Let\ x = 0.67\ \ in \quad 3x^2 = 2x$; $3 \cdot 0.67^2 \overset{?}{=} 2 \cdot 0.67$; $3 \cdot 0.448 \overset{?}{=} 1.34$; $1.34 = 1.34$

Check No. 2: $3x^2 - 2x \overset{?}{=} (x+0)(x-0.67)$; $3x^2 - 2x \overset{?}{=} (x \cdot x) + (-0.67 \cdot x) + (0 \cdot x) + (0 \cdot -0.67)$

; $3x^2 - 2x \overset{?}{=} x^2 - 0.67x + 0 + 0$; $3x^2 - 2x \overset{?}{=} x^2 - 0.67x$; $3x^2 - 2x \overset{?}{=} 3\left(x^2 - 0.67x\right)$

; $3x^2 - 2x = 3x^2 - 2x$

Therefore, the equation $3x^2 - 2x = 0$ can be factored to $(x+0)(x-0.67) = 0$ which is the same as $\boldsymbol{x(x - 0.67) = 0}$. Note that if both sides of the equation are multiplied by 3 we obtain $3 \cdot x(x - 0.67) = 0 \cdot 3$; $3x^2 - 2x = 0$ which is the same as the original equation.

Similar to the examples presented in Section 3.3 Case II in the Mastering Algebra – Intermediate Level book, the steps in solving the following class of quadratic equations is very similar, if not identical, to the previous problems solved in this section. However, in the following set of examples to ensure proper factorization, we need to accurately match the given coefficients of x^2, x, and the constant term with the coefficient and the constant term of the standard quadratic equation $ax^2 + bx + c = 0$. For example, given the quadratic equation $10x^2 - 14xy - 12y^2 = 0$ we know that $a = 10$, $b = -14y$, and $c = -12y^2$. Once this equality is established, then the remaining steps are identical to the steps used in solving the previous problems. To further illustrate this point the same examples that were used in Section 3.3 Case II in the Mastering Algebra – Intermediate Level book, i.e., examples 3.3-41 through 3.3-44 are solved below. However, the method used here is the Quadratic Formula method as opposed to the Trail and Error method, which was used in Section 3.3 of the book.

Example 1.2-25:

Solve $6x^2 + 10xy + 4y^2 = 0$ (x is variable and y is constant).

Solution:

First - Simplify the equation, i.e., $6x^2 + 10xy + 4y^2 = 0$; $2\left(3x^2 + 5xy + 2y^2\right) = 0$; $3x^2 + 5xy + 2y^2 = 0$

Second - Write the equation in standard form, i.e., write $6x^2 + 10xy + 4y^2 = 0$ as

$$6x^2 + (10y)x + 4y^2 = 0 .$$

Third - Equate the coefficient of the standard quadratic equation with the given equation, i.e., let $a = 6$, $b = 10y$, and $c = 4y^2$.

Fourth - Use the quadratic formula to solve the equation, i.e., given

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-10y\pm\sqrt{(10y)^2-4\times6\times4y^2}}{2\times6}$; $x=\frac{-10y\pm\sqrt{100y^2-96y^2}}{12}$; $x=\frac{-10y\pm\sqrt{4y^2}}{12}$; $x=\frac{-10y\pm\sqrt{2^2y^2}}{12}$; $x=\frac{-10y\pm2y}{12}$. Therefore:

I. $x=\frac{-10y+2y}{12}$; $x=-\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}y$; $x=-\frac{2}{3}y$; $\boldsymbol{x=-0.67y}$

II. $x=\frac{-10y-2y}{12}$; $x=-\frac{\cancel{12}}{\cancel{12}}y$; $\boldsymbol{x=-y}$

Thus, the solution set is $\{\boldsymbol{-y,-0.67y}\}$.

Fifth - Check the answer by substituting the solutions into the original equation.

I. *Let* $x=-0.67y$ *in* $3x^2+5xy+2y^2=0$; $3\cdot(-0.67y)^2+5\cdot(-0.67y)\cdot y+2y^2\overset{?}{=}0$; $3\times0.45y^2-3.35y^2+2y^2\overset{?}{=}0$; $1.35y^2-3.35y^2+2y^2\overset{?}{=}0$; $(1.35+2)y^2-3.35y^2\overset{?}{=}0$; $3.35y^2-3.35y^2\overset{?}{=}0$; $0=0$

II. *Let* $x=-y$ *in* $3x^2+5xy+2y^2=0$; $3\cdot(-y)^2+5\cdot(-y)\cdot y+2y^2\overset{?}{=}0$; $3y^2-5y^2+2y^2\overset{?}{=}0$; $(3+2)y^2-5y^2\overset{?}{=}0$; $5y^2-5y^2\overset{?}{=}0$; $0=0$

Therefore, the equation $3x^2+5xy+2y^2=0$ can be factored to $\boldsymbol{(x+0.67y)(x+y)=0}$ which is the same as $\left(x+\frac{2}{3}y\right)(2x+2y)=0$; $(3x+2y)(2x+2y)=0$. (Compare this answer with the result obtained in example 3.3-41 in the Mastering Algebra – Intermediate Level book.)

Sixth - Check the answer using the FOIL method.

$(x+0.67y)(x+y)=0$; $x\cdot x+x\cdot y+0.67y\cdot x+0.67y\cdot y=0$; $x^2+xy+0.67xy+0.67y^2=0$; $x^2+(1+0.67)xy+0.67y^2=0$; $x^2+1.67xy+0.67y^2=0$. Let's multiply both sides of the equation by 6, i.e., $6\cdot\left(x^2+1.67xy+0.67y^2\right)=6\cdot0$; $6x^2+10xy+4y^2=0$ which is the same as the original equation.

Example 1.2-26 A:

Solve $2x^2-19xy+35y^2=0$ (x is variable and y is constant).

Solution:

First - The equation is already in its simplest from.

Second - Write the equation in standard form, i.e., write $2x^2-19xy+35y^2=0$ as $2x^2+(-19y)x+35y^2=0$.

Third - Equate the coefficient of the standard quadratic equation with the given equation, i.e., let $a=2$, $b=-19y$, and $c=35y^2$.

Fourth - Use the quadratic formula to solve the equation, i.e., given

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \dfrac{-(-19y) \pm \sqrt{(-19y)^2 - 4 \times 2 \times 35y^2}}{2 \times 2}$; $x = \dfrac{19y \pm \sqrt{361y^2 - 280y^2}}{4}$

; $x = \dfrac{19y \pm \sqrt{81y^2}}{4}$; $x = \dfrac{19y \pm \sqrt{9^2 y^2}}{4}$; $x = \dfrac{19y \pm 9y}{4}$. Therefore:

I. $x = \dfrac{19y + 9y}{4}$; $x = \dfrac{\cancel{28}}{\cancel{4}} y$; $x = \dfrac{7}{1} y$; $\boldsymbol{x = 7y}$ II. $x = \dfrac{19y - 9y}{4}$; $x = \dfrac{\cancel{10}}{\cancel{4}} y$; $\boldsymbol{x = \dfrac{5}{2} y}$

Thus, the solution set is $\left\{ \boldsymbol{\dfrac{5}{2} y, 7y} \right\}$.

Fifth - Check the answer by substituting the solutions into the original equation.

I. *Let* $x = -7y$ *in* $2x^2 - 19xy + 35y^2 = 0$; $2 \cdot (7y)^2 - 19 \cdot 7y \cdot y + 35y^2 \overset{?}{=} 0$; $2 \cdot 49y^2 - 133y^2 + 35y^2 \overset{?}{=} 0$

; $98y^2 - 133y^2 + 35y^2 \overset{?}{=} 0$; $(98 - 133)y^2 + 35y^2 \overset{?}{=} 0$; $-35y^2 + 35y^2 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = \dfrac{5}{2} y$ *in* $2x^2 - 19xy + 35y^2 = 0$; $2 \cdot \left(\dfrac{5}{2} y \right)^2 - 19 \cdot \left(\dfrac{5}{2} y \right) \cdot y + 35y^2 \overset{?}{=} 0$

; $\cancel{2} \cdot \dfrac{25}{\underset{2}{\cancel{4}}} y^2 - 19 \cdot \dfrac{5}{2} y \cdot y + 35y^2 \overset{?}{=} 0$; $\dfrac{25}{2} y^2 - \dfrac{95}{2} y^2 + 35y^2 \overset{?}{=} 0$; $\left(\dfrac{25}{2} - \dfrac{95}{2} \right) y^2 + 35y^2 \overset{?}{=} 0$

; $\left(\dfrac{25 - 95}{2} \right) y^2 + 35y^2 \overset{?}{=} 0$; $-\dfrac{\overset{35}{\cancel{70}}}{\cancel{2}} y^2 + 35y^2 \overset{?}{=} 0$; $-35y^2 + 35y^2 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $2x^2 - 19xy + 35y^2$ can be factored to $\boldsymbol{(x - 7y)\left(x - \dfrac{5}{2} y \right) = 0}$ which is the same as $(x - 7y)(2x - 5y) = 0$. (Compare this answer with the result obtained in example 3.3-42A in the Mastering Algebra – Intermediate Level book.)

Sixth - Check the answer using the FOIL method.

$(x - 7y)(2x - 5y) = 0$; $x \cdot 2x - x \cdot 5y - 7y \cdot 2x - 7y \cdot (-5y) = 0$; $2x^2 - 5xy - 14xy + 35y^2 = 0$

; $2x^2 + (-5 - 14)xy + 35y^2 = 0$; $2x^2 - 19xy + 35y^2 = 0$ which is the same as the original equation.

Let's rework this problem. However, this time let y be the variable and x be the constant as follows:

Example 1.2-26 B:

Solve $2x^2 - 19xy + 35y^2 = 0$ (y is variable and x is constant).

Solution:

First - Write the equation in standard form, i.e., write $2x^2 - 19xy + 35y^2 = 0$ as

$35y^2 + (-19x)y + 2x^2 = 0$.

Second - Equate the coefficient of the standard quadratic equation with the given equation, i.e., let $a = 35$, $b = -19x$, and $c = 2x^2$.

Third - Use the quadratic formula to solve the equation, i.e., given

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $y = \dfrac{-(-19x) \pm \sqrt{(-19x)^2 - 4 \times 35 \times 2x^2}}{2 \times 35}$; $y = \dfrac{19x \pm \sqrt{361x^2 - 280x^2}}{70}$

; $y = \dfrac{19x \pm \sqrt{81x^2}}{70}$; $y = \dfrac{19x \pm \sqrt{9^2 x^2}}{70}$; $y = \dfrac{19x \pm 9x}{70}$. Therefore:

I. $y = \frac{19x + 9x}{70}$; $y = \frac{\overset{14}{\cancel{28}}}{\underset{35}{\cancel{70}}}x$; $y = \frac{14}{35}x$; $\boldsymbol{y = 0.4x}$ II. $y = \frac{19x - 9x}{70}$; $y = \frac{\cancel{10}}{\underset{7}{\cancel{70}}}x$; $\boldsymbol{y = \frac{1}{7}x}$

Thus, the solution set is $\left\{\boldsymbol{\frac{1}{7}x, 0.4x}\right\}$.

Fourth - Check the answer by substituting the solutions into the original equation.

I. *Let* $y = 0.4x$ *in* $2x^2 - 19xy + 35y^2 = 0$; $2x^2 - 19x\cdot(0.4x) + 35\cdot(0.4x)^2 \overset{?}{=} 0$

; $2x^2 - 7.6x^2 + 35\cdot 0.16x^2 \overset{?}{=} 0$; $2x^2 - 7.6x^2 + 5.6x^2 \overset{?}{=} 0$; $2x^2 + (-7.6 + 5.6)x^2 \overset{?}{=} 0$; $2x^2 - 2x^2 \overset{?}{=} 0$

; $0 = 0$

II. *Let* $y = \frac{1}{7}x$ *in* $2x^2 - 19xy + 35y^2 = 0$; $2x^2 - 19x\cdot\left(\frac{1}{7}x\right) + 35\cdot\left(\frac{1}{7}x\right)^2 \overset{?}{=} 0$

; $\cancel{2}\cdot\frac{25}{\underset{2}{\cancel{4}}}y^2 - 19\cdot\frac{5}{2}y\cdot y + 35y^2 \overset{?}{=} 0$; $\frac{25}{2}y^2 - \frac{95}{2}y^2 + 35y^2 \overset{?}{=} 0$; $\left(\frac{25}{2} - \frac{95}{2}\right)y^2 + 35y^2 \overset{?}{=} 0$

; $2x^2 - \frac{19}{7}x^2 + 35\cdot\frac{1}{49}x^2 \overset{?}{=} 0$; $2x^2 - 2.71x^2 + 0.71x^2 \overset{?}{=} 0$; $2x^2 + (-2.71 + 0.71)x^2 \overset{?}{=} 0$

; $2x^2 - 2x^2 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $35y^2 - 19xy + 2x^2 = 0$ can be factored to $\boldsymbol{(y - 0.4x)\left(y - \frac{1}{7}x\right) = 0}$ which is the same as $\left(y - \frac{4}{10}x\right)\cdot\left(y - \frac{1}{7}x\right) = 0$; $\left(y - \frac{2}{5}x\right)\cdot\left(y - \frac{1}{7}x\right) = 0$; $(5y - 2x)\cdot(7y - x) = 0$.

(Compare this answer with the result obtained in example 3.3-42B in the Mastering Algebra – Intermediate Level book.)

Fifth - Check the answer using the FOIL method.

$(y - 0.4x)\left(y - \frac{1}{7}x\right) = 0$; $\left(y - \frac{14}{35}x\right)\left(y - \frac{1}{7}x\right) = 0$; $\left(\frac{35y - 14x}{35}\right)\left(\frac{7y - x}{7}\right) = 0$; $\frac{(35y - 14x)\cdot(7y - x)}{35\cdot 7} = 0$

; $\frac{(35y - 14x)\cdot(7y - x)}{245} = \frac{0}{1}$; $[(35y - 14x)\cdot(7y - x)]\cdot 1 = 245\cdot 0$; $(35y - 14x)\cdot(7y - x) = 0$

; $245y^2 - 35xy - 98xy + 14x^2 = 0$; $245y^2 + (-35 - 98)xy + 14x^2 = 0$; $245y^2 - 133xy + 14x^2 = 0$

; $\frac{245y^2 - 133xy + 14x^2}{7} = \frac{0}{7}$; $35y^2 - 19xy + 2x^2 = 0$ which is the same as the original equation.

Example 1.2-27:

Solve $3r^2 + 11rs + 10s^2 = 0$ (r is variable and s is constant).

Solution:

First - Write the equation in standard form, i.e., write $3r^2 + 11rs + 10s^2 = 0$ as

$3r^2 + (11s)r + 10s^2 = 0$.

Second - Equate the coefficient of the standard quadratic equation with the given equation, i.e., let $a = 3$, $b = 11s$, and $c = 10s^2$.

Third - Use the quadratic formula to solve the equation, i.e., given

$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $r=\frac{-11s\pm\sqrt{(11s)^2-4\times3\times10s^2}}{2\times3}$; $r=\frac{-11s\pm\sqrt{121s^2-120s^2}}{6}$; $r=\frac{-11s\pm\sqrt{s^2}}{6}$; $r=\frac{-11s\pm s}{6}$. Therefore:

I. $r=\frac{-11s+s}{6}$; $r=-\frac{\overset{5}{\cancel{10}}s}{\underset{3}{\cancel{6}}}$; $r=-\frac{5}{3}s$; $\boldsymbol{r=-1.67s}$

II. $r=\frac{-11s-s}{6}$; $r=-\frac{\overset{2}{\cancel{12}}}{\cancel{6}}s$; $r=-\frac{2}{1}s$; $\boldsymbol{r=-2s}$ and the solution set is $\{\boldsymbol{-2s,-1.67s}\}$.

Fourth - Check the answer by substituting the solutions into the original equation.

I. *Let* $r=-1.67s$ *in* $3r^2+11rs+10s^2=0$; $3\cdot(-1.67s)^2+11\cdot(-1.67s)\cdot s+10s^2\overset{?}{=}0$; $3\cdot\left(2.79s^2\right)-18.37s^2+10s^2\overset{?}{=}0$; $8.37s^2-18.37s^2+10s^2\overset{?}{=}0$; $(8.37+10)s^2-18.37s^2\overset{?}{=}0$; $18.37s^2-18.37s^2\overset{?}{=}0$; $0=0$

II. *Let* $r=-2s$ *in* $3r^2+11rs+10s^2=0$; $3\cdot(-2s)^2+11\cdot(-2s)\cdot s+10s^2\overset{?}{=}0$; $3\cdot4s^2-22s^2+10s^2\overset{?}{=}0$; $12s^2-22s^2+10s^2\overset{?}{=}0$; $(12+10)s^2-22s^2\overset{?}{=}0$; $22s^2-22s^2\overset{?}{=}0$; $0=0$

Therefore, the equation $3r^2+11rs+10s^2=0$ can be factored to $\boldsymbol{(r+1.67s)(r+2s)=0}$ which is the same as $\left(r+\frac{5}{3}s\right)(r+2s)=0$; $(3r+5s)(r+2s)=0$. (Compare this answer with the result obtained in example 3.3-43 in the Mastering Algebra – Intermediate Level book.)

Fifth - Check the answer using the FOIL method.

$(r+1.67s)(r+2s)=0$; $r\cdot r+r\cdot2s+1.67s\cdot r+1.67s\cdot2s=0$; $r^2+2rs+1.67rs+3.34s^2=0$; $r^2+(2+1.67)rs+3.34s^2=0$; $r^2+3.67rs+3.34s^2=0$. Let's multiply both sides of the equation by 3, i.e., $3\cdot\left(r^2+3.67rs+3.34s^2\right)=3\cdot0$; $3r^2+11rs+10r^2=0$ which is the same as the original equation.

Example 1.2-28:

Solve $21n^2+41mn+10m^2=0$ (n is variable and m is constant).

Solution:

First - Write the equation in standard form, i.e., write $21n^2+41mn+10m^2=0$ as $21n^2+(41m)n+10m^2=0$.

Second - Equate the coefficient of the standard quadratic equation with the given equation, i.e., let $a=21$, $b=41m$, and $c=10m^2$.

Third - Use the quadratic formula to solve the equation, i.e., given:

$n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $n=\frac{-41m\pm\sqrt{(41m)^2-4\times21\times10m^2}}{2\times21}$; $n=\frac{-41m\pm\sqrt{1681m^2-840m^2}}{42}$; $n=\frac{-41m\pm\sqrt{841m^2}}{42}$; $n=\frac{-41m\pm\sqrt{29^2m^2}}{42}$; $n=\frac{-41m\pm29m}{42}$. Therefore:

I. $n = \dfrac{-41m+29m}{42}$; $n = -\dfrac{\overset{6}{\cancel{12}}}{\underset{21}{\cancel{42}}} m$; $n = -\dfrac{6}{21} m$; $\boldsymbol{n = -0.28m}$

II. $n = \dfrac{-41m-29m}{42}$; $n = -\dfrac{\overset{35}{\cancel{70}}}{\underset{21}{\cancel{42}}} m$; $n = -\dfrac{35}{21} m$; $\boldsymbol{n = -1.66m}$

Thus, the solution set is $\{\boldsymbol{-0.28m, -1.66m}\}$.

Fourth - Check the answer by substituting the solutions into the original equation.

I. *Let* $n = -0.28m$ *in* $21n^2 + 41mn + 10m^2 = 0$; $21 \cdot (-0.28m)^2 + 41m \cdot (-0.28m) + 10m^2 \overset{?}{=} 0$

; $21 \cdot \left(0.08m^2\right) - 11.48m^2 + 10m^2 \overset{?}{=} 0$; $1.6m^2 - 11.6m^2 + 10m^2 \overset{?}{=} 0$; $(1.6+10)m^2 - 11.6m^2 \overset{?}{=} 0$

; $11.6m^2 - 11.6m^2 \overset{?}{=} 0$; $0 = 0$

II. *Let* $n = -1.66m$ *in* $21n^2 + 41mn + 10m^2 = 0$; $21 \cdot (-1.66m)^2 + 41m \cdot (-1.66m) + 10m^2 \overset{?}{=} 0$

; $21 \cdot \left(2.75m^2\right) - 68m^2 + 10m^2 \overset{?}{=} 0$; $58m^2 - 68m^2 + 10m^2 \overset{?}{=} 0$; $(58+10)m^2 - 68m^2 \overset{?}{=} 0$

; $68m^2 - 68m^2 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $21n^2 + 41mn + 10m^2 = 0$ can be factored to $\boldsymbol{(n + 0.28m)(n + 1.66m) = 0}$ which is the same as $\left(n + \frac{2}{7}m\right)\left(n + \frac{5}{3}m\right) = 0$; $(7n + 2m)(3n + 5m) = 0$. (Compare this answer with the result obtained in example 3.3-44 in the Mastering Algebra – Intermediate Level book.)

Fifth - Check the answer using the FOIL method.

$(n + 0.28m)(n + 1.66m) = 0$; $n \cdot n + n \cdot 1.66m + 0.28m \cdot n + 0.28m \cdot 1.66m = 0$

; $n^2 + 1.66mn + 0.28mn + 0.46m^2 = 0$; $n^2 + (1.66 + 0.28)mn + 0.46m^2 = 0$; $n^2 + 1.94mn + 0.46m^2 = 0$.

Let's multiply both sides of the equation by 21, i.e., $21 \cdot \left(n^2 + 1.94mn + 0.46m^2\right) = 21 \cdot 0$

; $21n^2 + 41mn + 10m^2 = 0$ which is the same as the original equation.

Note that since the solutions are rounded off to the first two digits, in some instances, we do not obtain an exact match with the coefficients of the original equation.

Practice Problems - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a \rangle 1$, Using the Quadratic Formula

Section 1.2 Case II Practice Problems - Use the quadratic formula to solve the following equations.

1. $4u^2 + 6u + 1 = 0$
2. $4w^2 + 10w = -3$
3. $6x^2 + 4x - 2 = 0$
4. $15y^2 + 3 = -14y$
5. $2x^2 - 5x + 3 = 0$
6. $2x^2 + xy - y^2 = 0$ x is variable
7. $6x^2 + 7x - 3 = 0$
8. $5x^2 = -3x$
9. $3x^2 + 4x + 5 = 0$
10. $-3y^2 + 13y + 10 = 0$

1.3 Solving Quadratic Equations Using the Square Root Property Method

Quadratic equations of the form $(ax+b)^2 = c$ are solved using a method known as the Square Root Property method where the square root of both sides of the equation are taken and the terms are simplified. Following show the steps as to how quadratic equations are solved using the Square Root property method:

Step 1 Take the square root of the left and the right hand side of the equation. Simplify the terms on both sides of the equation.

Step 2 Solve for the values of x. Check the answers by substituting the x values into the original equation.

Step 3 Write the equation in its factored form.

Examples with Steps

The following examples show the steps as to how equations of the form $(ax+b)^2 = c$ are solved using the Square Root Property method:

Example 1.3-1

Solve the quadratic equation $(x+4)^2 = 36$.

Solution:

Step 1 $\boxed{(x+4)^2 = 36}$; $\boxed{\sqrt{(x+4)^2} = \pm\sqrt{36}}$; $\boxed{\sqrt{(x+4)^2} = \pm\sqrt{6^2}}$; $\boxed{x+4 = \pm 6}$

Step 2 Separate $x+4 = \pm 6$ into two equations.

I. $\boxed{x+4 = +6}$; $\boxed{x = +6-4}$; $\boxed{\mathbf{x = 2}}$

II. $\boxed{x+4 = -6}$; $\boxed{x = -6-4}$; $\boxed{\mathbf{x = -10}}$

Thus, the solution set is $\{\mathbf{-10, 2}\}$.

Check: I. *Let* $x = 2$ *in* $(x+4)^2 = 36$; $(2+4)^2 \stackrel{?}{=} 36$; $6^2 \stackrel{?}{=} 36$; $36 = 36$

II. *Let* $x = -10$ *in* $(x+4)^2 = 36$; $(-10+4)^2 \stackrel{?}{=} 36$; $(-6)^2 \stackrel{?}{=} 36$; $36 = 36$

Step 3 Therefore, the equation $(x+4)^2 = 36$ can be factored to $\mathbf{(x-2)(x+10) = 0}$.

Example 1.3-2

Solve the quadratic equation $(x-2)^2 = 25$.

Solution:

Step 1 $\boxed{(x-2)^2 = 25}$; $\boxed{\sqrt{(x-2)^2} = \pm\sqrt{25}}$; $\boxed{\sqrt{(x-2)^2} = \pm\sqrt{5^2}}$; $\boxed{x-2 = \pm 5}$

Step 2 Separate $x-2 = \pm 5$ into two equations.

I. $\boxed{x-2 = +5}$; $\boxed{x = 5+2}$; $\boxed{\mathbf{x = 7}}$

II. $\boxed{x-2 = -5}$; $\boxed{x = -5+2}$; $\boxed{\mathbf{x = -3}}$

Thus, the solution set is $\{\mathbf{-3, 7}\}$.

Check: I. *Let* $x = 7$ *in* $(x-2)^2 = 25$; $(7-2)^2 \overset{?}{=} 25$; $5^2 \overset{?}{=} 25$; $25 = 25$

II. *Let* $x = -3$ *in* $(x-2)^2 = 25$; $(-3-2)^2 \overset{?}{=} 25$; $(-5)^2 \overset{?}{=} 25$; $25 = 25$

Step 3 Therefore, the equation $(x-2)^2 = 25$ can be factored to $(\boldsymbol{x-7})(\boldsymbol{x+3}) = \mathbf{0}$.

Example 1.3-3

Solve the quadratic equation $(x+2)^2 = 8$.

Solution:

Step 1 $\boxed{(x+2)^2 = 8}$; $\boxed{\sqrt{(x+2)^2} = \pm\sqrt{8}}$; $\boxed{\sqrt{(x+2)^2} = \pm\sqrt{4 \cdot 2}}$; $\boxed{x+2 = \pm 2\sqrt{2}}$

Step 2 Separate $x+2 = \pm 2\sqrt{2}$ into two equations.

I. $\boxed{x+2 = +2\sqrt{2}}$; $\boxed{x+2-2 = -2+2\sqrt{2}}$; $\boxed{x+0 = -2+2\sqrt{2}}$; $\boxed{\boldsymbol{x = -2+2\sqrt{2}}}$

II. $\boxed{x+2 = -2\sqrt{2}}$; $\boxed{x+2-2 = -2-2\sqrt{2}}$; $\boxed{x+0 = -2-2\sqrt{2}}$; $\boxed{\boldsymbol{x = -2-2\sqrt{2}}}$

Thus, the solution set is $\{\mathbf{-2-2\sqrt{2},\ -2+2\sqrt{2}}\}$.

Check: I. *Let* $x = -2+2\sqrt{2}$ *in* $(x+2)^2 = 8$; $(-\cancel{2}+2\sqrt{2}+\cancel{2})^2 \overset{?}{=} 8$; $(2\sqrt{2})^2 \overset{?}{=} 8$; $4 \cdot 2 \overset{?}{=} 8$; $8 = 8$

II. *Let* $x = -2-2\sqrt{2}$ *in* $(x+2)^2 = 8$; $(-\cancel{2}-2\sqrt{2}+\cancel{2})^2 \overset{?}{=} 8$; $(-2\sqrt{2})^2 \overset{?}{=} 8$; $4 \cdot 2 \overset{?}{=} 8$; $8 = 8$

Step 3 Thus, the equation $(x+2)^2 = 8$ can be factored to $(\boldsymbol{x+2-2\sqrt{2}})(\boldsymbol{x+2+2\sqrt{2}})$.

Example 1.3-4

Solve the quadratic equation $(2x-4)^2 = 16$.

Solution:

Step 1 $\boxed{(2x-4)^2 = 16}$; $\boxed{\sqrt{(2x-4)^2} = \pm\sqrt{16}}$; $\boxed{\sqrt{(2x-4)^2} = \pm\sqrt{4^2}}$; $\boxed{2x-4 = \pm 4}$

Step 2 Separate $2x-4 = \pm 4$ into two equations.

I. $\boxed{2x-4 = +4}$; $\boxed{2x = 4+4}$; $\boxed{2x = 8}$; $\boxed{x = \frac{\overset{4}{\cancel{8}}}{\cancel{2}}}$; $\boxed{x = \frac{4}{1}}$; $\boxed{\boldsymbol{x = 4}}$

II. $\boxed{2x-4 = -4}$; $\boxed{2x = -4+4}$; $\boxed{2x = 0}$; $\boxed{x = \frac{0}{2}}$; $\boxed{\boldsymbol{x = 0}}$

Thus, the solution set is $\{\mathbf{0, 4}\}$.

Check: I. *Let* $x = 4$ *in* $(2x-4)^2 = 16$; $(2 \cdot 4 - 4)^2 \overset{?}{=} 16$; $(8-4)^2 \overset{?}{=} 16$; $4^2 \overset{?}{=} 16$; $16 = 16$

II. *Let* $x = 0$ *in* $(2x-4)^2 = 16$; $(2 \cdot 0 - 4)^2 \overset{?}{=} 16$; $(0-4)^2 \overset{?}{=} 16$; $(-4)^2 \overset{?}{=} 16$

$; \ 16 = 16$

Step 3 Thus, the equation $(2x-4)^2 = 16$ can be factored to $(x-4)(x+0) = 0$ which is the same as $\boldsymbol{x(x-4) = 0}$

Example 1.3-5

Solve the quadratic equation $\left(y+\frac{2}{3}\right)^2 = \frac{4}{9}$.

Solution:

Step 1 $\boxed{\left(y+\frac{2}{3}\right)^2 = \frac{4}{9}}$; $\boxed{\sqrt{\left(y+\frac{2}{3}\right)^2} = \pm\sqrt{\frac{4}{9}}}$; $\boxed{\sqrt{\left(y+\frac{2}{3}\right)^2} = \pm\sqrt{\frac{2^2}{3^2}}}$; $\boxed{y+\frac{2}{3} = \pm\frac{2}{3}}$

Step 2 Separate $y+\frac{2}{3} = \pm\frac{2}{3}$ into two equations

I. $\boxed{y+\frac{2}{3} = +\frac{2}{3}}$; $\boxed{y = -\frac{2}{3}-\frac{2}{3}}$; $\boxed{y = \frac{-2-2}{3}}$; $\boxed{\boldsymbol{y = -\frac{4}{3}}}$

II. $\boxed{y+\frac{2}{3} = -\frac{2}{3}}$; $\boxed{y = -\frac{2}{3}+\frac{2}{3}}$; $\boxed{y = \frac{-2+2}{3}}$; $\boxed{y = \frac{0}{3}}$; $\boxed{\boldsymbol{y = 0}}$

Thus, the solution set is $\left\{\boldsymbol{0, -\frac{4}{3}}\right\}$.

Check: I. *Let* $y = -\frac{4}{3}$ *in* $\left(y+\frac{2}{3}\right)^2 = \frac{4}{9}$; $\left(-\frac{4}{3}+\frac{2}{3}\right)^2 \stackrel{?}{=} \frac{4}{9}$; $\left(\frac{-4+2}{3}\right)^2 \stackrel{?}{=} \frac{4}{9}$; $\left(\frac{-2}{3}\right)^2 \stackrel{?}{=} \frac{4}{9}$; $\frac{4}{9} = \frac{4}{9}$

II. *Let* $y = 0$ *in* $\left(y+\frac{2}{3}\right)^2 = \frac{4}{9}$; $\left(0+\frac{2}{3}\right)^2 \stackrel{?}{=} \frac{4}{9}$; $\left(\frac{2}{3}\right)^2 \stackrel{?}{=} \frac{4}{9}$; $\frac{4}{9} = \frac{4}{9}$

Step 3 Therefore, the equation $\left(y+\frac{2}{3}\right)^2 = \frac{4}{9}$ can be factored to $\left(y+\frac{4}{3}\right)(y+0) = 0$; $y\left(y+\frac{4}{3}\right) = 0$ which is the same as $\boldsymbol{y(3y+4) = 0}$.

Additional Examples - Solving Quadratic Equations Using the Square Root Property Method

The following examples further illustrate how to solve quadratic equations using the Square Root Property method:

Example 1.3-6

Solve the quadratic equation $(6u-3)^2 = 25$ using the Square Root Property method.

Solution:

$\boxed{(6u-3)^2 = 25}$; $\boxed{\sqrt{(6u-3)^2} = \pm\sqrt{25}}$; $\boxed{\sqrt{(6u-3)^2} = \pm\sqrt{5^2}}$; $\boxed{6u-3 = \pm 5}$

Therefore, the two solutions are:

I. $\boxed{6u-3 = +5}$; $\boxed{6u = 5+3}$; $\boxed{6u = 8}$; $\boxed{u = \frac{\cancel{8}^{\,4}}{\cancel{6}_{\,3}}}$; $\boxed{\boldsymbol{u = \frac{4}{3}}}$

II. $\boxed{6u-3=-5}$; $\boxed{6u=-5+3}$; $\boxed{6u=-2}$; $\boxed{u=-\frac{\cancel{2}}{\cancel{6}_3}}$; $\boxed{\boldsymbol{u=-\frac{1}{3}}}$

Thus, the solution set is $\left\{-\frac{\mathbf{1}}{\mathbf{3}}, \frac{\mathbf{4}}{\mathbf{3}}\right\}$.

Check: I. $Let\ u=\frac{4}{3}\ in \quad (6u-3)^2=25$; $\left(6\cdot\frac{4}{3}-3\right)^2 \stackrel{?}{=} 25$; $\left(\frac{\cancel{24}^{8}}{\cancel{3}}-3\right)^2 \stackrel{?}{=} 25$; $\left(\frac{8}{1}-3\right)^2 \stackrel{?}{=} 25$; $(8-3)^2 \stackrel{?}{=} 25$

; $5^2 \stackrel{?}{=} 25$; $25=25$

II. $Let\ u=-\frac{1}{3}\ in \quad (6u-3)^2=25$; $\left[\left(6\cdot-\frac{1}{3}\right)-3\right]^2 \stackrel{?}{=} 25$; $\left(-\frac{\cancel{6}^{2}}{\cancel{3}}-3\right)^2 \stackrel{?}{=} 25$; $\left(-\frac{2}{1}-3\right)^2 \stackrel{?}{=} 25$

; $(-2-3)^2 \stackrel{?}{=} 25$; $(-5)^2 \stackrel{?}{=} 25$; $25=25$

Therefore, the equation $(6u-3)^2=25$ can be factored to $\left(u-\frac{4}{3}\right)\left(u+\frac{1}{3}\right)=0$ which is the same as $\boldsymbol{(3u-4)(3u+1)=0}$.

Example 1.3-7

Solve the quadratic equation $(5y+3)^2=15$ using the Square Root Property method.

Solution:

$\boxed{(5y+3)^2=15}$; $\boxed{\sqrt{(5y+3)^2}=\pm\sqrt{15}}$; $\boxed{5y+3=\pm\sqrt{15}}$ Therefore, the two solutions are:

I. $\boxed{5y+3=+\sqrt{15}}$; $\boxed{5y=\sqrt{15}-3}$; $\boxed{\boldsymbol{y=\frac{\sqrt{15}-3}{5}}}$ II. $\boxed{5y+3=-\sqrt{15}}$; $\boxed{5y=-\sqrt{15}-3}$; $\boxed{\boldsymbol{y=-\frac{\sqrt{15}+3}{5}}}$

Thus, the solution set is $\left\{-\frac{\boldsymbol{\sqrt{15}+3}}{\mathbf{5}}, \frac{\boldsymbol{\sqrt{15}-3}}{\mathbf{5}}\right\}$.

Check: I. $Let\ y=\frac{\sqrt{15}-3}{5}\ in \quad (5y+3)^2=15$; $\left(\cancel{5}\cdot\frac{\sqrt{15}-3}{\cancel{5}}+3\right)^2 \stackrel{?}{=} 15$; $\left(\sqrt{15}-\cancel{3}+\cancel{3}\right)^2 \stackrel{?}{=} 15$; $\left(\sqrt{15}\right)^2 \stackrel{?}{=} 15$

; $15=15$

II. $Let\ y=-\frac{\sqrt{15}+3}{5}\ in \quad (5y+3)^2=15$; $\left[\left(\cancel{5}\cdot-\frac{\sqrt{15}+3}{\cancel{5}}\right)+3\right]^2 \stackrel{?}{=} 15$; $\left[\left(-\sqrt{15}-3\right)+3\right]^2 \stackrel{?}{=} 15$

; $\left(-\sqrt{15}-\cancel{3}+\cancel{3}\right)^2 \stackrel{?}{=} 15$; $\left(-\sqrt{15}\right)^2 \stackrel{?}{=} 15$; $15=15$

Therefore, the equation $(5y+3)^2=15$ can be factored to $\left(y-\frac{\sqrt{15}-3}{5}\right)\left(y+\frac{\sqrt{15}+3}{5}\right)=0$ which is the same as $\boldsymbol{(y-0.175)(y+1.375)=0}$; $y^2+1.2y-0.24=0$; $25y^2+30y-6=0$, or $(5y+3)^2=15$.

Example 1.3-8

Solve the quadratic equation $(2w-4)^2=1$ using the Square Root Property method.

Solution:

$\boxed{(2w-4)^2=1}$; $\boxed{\sqrt{(2w-4)^2}=\pm\sqrt{1}}$; $\boxed{\sqrt{(2w-4)^2}=\pm 1}$; $\boxed{2w-4=\pm 1}$ Therefore, the two solutions are:

I. $\boxed{2w-4=+1}$; $\boxed{2w=4+1}$; $\boxed{2w=5}$; $\boxed{\boldsymbol{w=\frac{5}{2}}}$ II. $\boxed{2w-4=-1}$; $\boxed{2w=4-1}$; $\boxed{2w=3}$; $\boxed{\boldsymbol{w=\frac{3}{2}}}$

Thus, the solution set is $\left\{\boldsymbol{\frac{3}{2},\frac{5}{2}}\right\}$.

Check: I. *Let* $w=\frac{5}{2}$ *in* $(2w-4)^2=1$; $\left(\cancel{2}\cdot\frac{5}{\cancel{2}}-4\right)^2\stackrel{?}{=}1$; $(5-4)^2\stackrel{?}{=}1$; $1^2\stackrel{?}{=}1$; $1=1$

II. *Let* $w=\frac{3}{2}$ *in* $(2w-4)^2=1$; $\left(\cancel{2}\cdot\frac{3}{\cancel{2}}-4\right)^2\stackrel{?}{=}1$; $(3-4)^2\stackrel{?}{=}1$; $(-1)^2\stackrel{?}{=}1$; $1=1$

Therefore, the equation $(2w-4)^2=1$ can be factored to $\left(w-\frac{5}{2}\right)\left(w-\frac{3}{2}\right)=0$ which is the same as $\boldsymbol{(2w-5)(2w-3)=0}$.

Example 1.3-9

Solve the quadratic equation $\left(x-\frac{1}{2}\right)^2=\frac{1}{16}$ using the Square Root Property method.

Solution:

$\boxed{\left(x-\frac{1}{2}\right)^2=\frac{1}{16}}$; $\boxed{\sqrt{\left(x-\frac{1}{2}\right)^2}=\pm\sqrt{\frac{1}{16}}}$; $\boxed{\sqrt{\left(x-\frac{1}{2}\right)^2}=\pm\sqrt{\frac{1}{4^2}}}$; $\boxed{x-\frac{1}{2}=\pm\frac{1}{4}}$

Therefore, the two solutions are:

I. $\boxed{x-\frac{1}{2}=+\frac{1}{4}}$; $\boxed{x=\frac{1}{4}+\frac{1}{2}}$; $\boxed{x=\frac{(1\cdot 2)+(1\cdot 4)}{2\cdot 4}}$; $\boxed{x=\frac{2+4}{8}}$; $\boxed{x=\frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}}}$; $\boxed{\boldsymbol{x=\frac{3}{4}}}$

II. $\boxed{x-\frac{1}{2}=-\frac{1}{4}}$; $\boxed{x=-\frac{1}{4}+\frac{1}{2}}$; $\boxed{x=\frac{(-1\cdot 2)+(1\cdot 4)}{2\cdot 4}}$; $\boxed{x=\frac{-2+4}{8}}$; $\boxed{x=\frac{\cancel{2}}{\underset{4}{\cancel{8}}}}$; $\boxed{\boldsymbol{x=\frac{1}{4}}}$

Thus, the solution set is $\left\{\boldsymbol{\frac{1}{4},\frac{3}{4}}\right\}$.

Check: I. *Let* $x=\frac{3}{4}$ *in* $\left(x-\frac{1}{2}\right)^2=\frac{1}{16}$; $\left(\frac{3}{4}-\frac{1}{2}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(\frac{(2\cdot 3)-(1\cdot 4)}{2\cdot 4}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(\frac{6-4}{8}\right)^2\stackrel{?}{=}\frac{1}{16}$

; $\left(\frac{\cancel{2}}{\underset{4}{\cancel{8}}}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(\frac{1}{4}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\frac{1}{4^2}\stackrel{?}{=}\frac{1}{16}$; $\frac{1}{16}=\frac{1}{16}$

II. *Let* $x=\frac{1}{4}$ *in* $\left(x-\frac{1}{2}\right)^2=\frac{1}{16}$; $\left(\frac{1}{4}-\frac{1}{2}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(\frac{(1\cdot 2)-(1\cdot 4)}{2\cdot 4}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(\frac{2-4}{8}\right)^2\stackrel{?}{=}\frac{1}{16}$

; $\left(-\frac{\cancel{2}}{\underset{4}{\cancel{8}}}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\left(-\frac{1}{4}\right)^2\stackrel{?}{=}\frac{1}{16}$; $\frac{1}{4^2}\stackrel{?}{=}\frac{1}{16}$; $\frac{1}{16}=\frac{1}{16}$

Therefore, the equation $\left(x-\frac{1}{2}\right)^2 = \frac{1}{16}$ can be factored to $\left(x-\frac{3}{4}\right)\left(x-\frac{1}{4}\right)=0$ which is the same as $\mathbf{(4x-3)(4x-1)=0}$.

Example 1.3-10

Solve the quadratic equation $(x+5)^2 = 49$ using the Square Root Property and the Quadratic Formula method.

Solution:

First Method - The Square Root Property method:

$\boxed{(x+5)^2 = 49}$; $\boxed{\sqrt{(x+5)^2} = \pm\sqrt{49}}$; $\boxed{\sqrt{(x+5)^2} = \pm\sqrt{7^2}}$; $\boxed{x+5=\pm 7}$ Therefore, the two solutions are:

I. $\boxed{x+5=+7}$; $\boxed{x=7-5}$; $\boxed{\mathbf{x=2}}$

II. $\boxed{x+5=-7}$; $\boxed{x=-7-5}$; $\boxed{\mathbf{x=-12}}$

Thus, the solution set is $\{\mathbf{-12, 2}\}$.

Check: I. *Let* $x=2$ *in* $(x+5)^2 = 49$; $(2+5)^2 \stackrel{?}{=} 49$; $7^2 \stackrel{?}{=} 49$; $49 = 49$

II. *Let* $x=-13$ *in* $(x+5)^2 = 49$; $(-12+5)^2 \stackrel{?}{=} 49$; $(-7)^2 \stackrel{?}{=} 49$; $49 = 49$

Therefore, the equation $(x+5)^2 = 49$ can be factored to $\mathbf{(x-2)(x+12)=0}$.

Second Method - The Quadratic Formula method:

Given the expression $(x+5)^2 = 49$, expand the left hand side of the equation and write the quadratic equation in its standard form, i.e.,

$\boxed{(x+5)^2 = 49}$; $\boxed{x^2+25+10x=49}$; $\boxed{x^2+10x+(25-49)=0}$; $\boxed{x^2+10x-24=0}$

Let: $\boxed{a=1}$, $\boxed{b=10}$, and $\boxed{c=-24}$. Then,

Given: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$; $\boxed{x=\frac{-10\pm\sqrt{10^2-(4\times 1\times -24)}}{2\times 1}}$; $\boxed{x=\frac{-10\pm\sqrt{100+96}}{2}}$; $\boxed{x=\frac{-10\pm\sqrt{196}}{2}}$

; $\boxed{x=\frac{-10\pm\sqrt{14^2}}{2}}$; $\boxed{x=\frac{-10\pm 14}{2}}$ Therefore, we can separate x into two equations:

I. $\boxed{x=\frac{-10+14}{2}}$; $\boxed{x=\frac{\overset{2}{\cancel{4}}}{\cancel{2}}}$; $\boxed{x=\frac{2}{1}}$; $\boxed{\mathbf{x=2}}$ II. $\boxed{x=\frac{-10-14}{2}}$; $\boxed{x=-\frac{\overset{12}{\cancel{24}}}{\cancel{2}}}$; $\boxed{x=-\frac{12}{1}}$; $\boxed{\mathbf{x=-12}}$

Thus, the solution set is $\{\mathbf{-12, 2}\}$.

The equation $(x+5)^2 = 49$ can be factored to $\mathbf{(x-2)(x+12)=0}$.

Note: As you may have already noticed, using the quadratic formula may not be a good choice since it requires more work and takes longer to solve. The key to solving quadratic

equations is selection of a method that is easiest to use. Further discussions on selection of a best method is addressed in Section 1.6.

Note that when $b = 0$ the quadratic equation $(ax+b)^2 = c$ reduces to $(ax)^2 = c$. The following examples show the steps as to how quadratic equations of the form $(ax)^2 = c$ are solved for cases where the coefficient of x is equal to or greater than one.

- For cases where $a = 1$, we can solve equations of the form $x^2 = c$ using the Square Root Property method in the following way:

  Example 1.3-11

  Solve $x^2 = 16$ using the Square Root Property method.

  Solution:

  First - Take the square root of both sides of the equation, i.e., $\sqrt{x^2} = \pm\sqrt{16}$

  Second - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{x = \pm 4}$. Therefore, the solution set is $\{-4, 4\}$ and the equation $x^2 = 16$ can be factored to $\boldsymbol{(x-4)(x+4) = 0}$.

  Check: I. $Let\ x = -4\ \ in\ \ \ x^2 = 16\ ;\ (-4)^2 \stackrel{?}{=} 16\ ;\ 16 = 16$

  II. $Let\ x = 4\ \ in\ \ \ x^2 = 16\ ;\ 4^2 \stackrel{?}{=} 16\ ;\ 16 = 16$

  Example 1.3-12

  Solve $w^2 = 5$ using the Square Root Property method.

  Solution:

  First - Take the square root of both sides of the equation, i.e., $\sqrt{w^2} = \pm\sqrt{5}$

  Second - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{w = \pm\sqrt{5}}$. Therefore, the solution set is $\{-\sqrt{5}, \sqrt{5}\}$ and the equation $w^2 = 5$ can be factored to $\boldsymbol{(w-\sqrt{5})(w+\sqrt{5}) = 0}$.

  Check: I. $Let\ w = -\sqrt{5}\ \ in\ \ \ w^2 = 5\ ;\ \left(-\sqrt{5}\right)^2 \stackrel{?}{=} 5\ ;\ \left(+5^{\frac{1}{2}}\right)^2 \stackrel{?}{=} 5\ ;\ 5^{\frac{1}{2}\times 2} \stackrel{?}{=} 5\ ;\ 5 = 5$

  II. $Let\ w = \sqrt{5}\ \ in\ \ \ w^2 = 5\ ;\ \left(\sqrt{5}\right)^2 \stackrel{?}{=} 5\ ;\ \left(5^{\frac{1}{2}}\right)^2 \stackrel{?}{=} 5\ ;\ 5^{\frac{1}{2}\times 2} \stackrel{?}{=} 5\ ;\ 5 = 5$

- For cases where $a \rangle 1$, we can solve equations of the form $(ax)^2 = c$ (which is the same as $kx^2 = c$, where $k = a^2$) using the Square Root Property method in the following way:

  Example 1.3-13

  Solve $3x^2 = 27$ using the Square Root Property method.

  Solution:

  First - Divide both sides of the equation by the coefficient x, i.e., $\frac{\cancel{3}x^2}{\cancel{3}} = \frac{\overset{9}{\cancel{27}}}{\cancel{3}}\ ;\ x^2 = \frac{9}{1}\ ;\ x^2 = 9$

  Second - Take the square root of both sides of the equation, i.e., $\sqrt{x^2} = \pm\sqrt{9}$

  Third - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{x = \pm 3}$

  Therefore, the solution set is $\{-3, 3\}$ and the equation $3x^2 = 27$ can be factored to $\boldsymbol{(x-3)(x+3) = 0}$.

Check: I. $Let\ x=-3\ in\quad 3x^2=27\ ;\ 3\cdot(-3)^2\overset{?}{=}27\ ;\ 3\cdot 9\overset{?}{=}27\ ;\ 27=27$

II. $Let\ x=3\ in\quad 3x^2=27\ ;\ 3\cdot 3^2\overset{?}{=}27\ ;\ 3\cdot 9\overset{?}{=}27\ ;\ 27=27$

Example 1.3-14

Solve $2y^2=9$ using the Square Root Property method.

Solution:

First - Divide both sides of the equation by the coefficient y, i.e., $\frac{\cancel{2}y^2}{\cancel{2}}=\frac{9}{2}\ ;\ y^2=\frac{9}{2}\ ;\ y^2=4.5$

Second - Take the square root of both sides of the equation, i.e., $\sqrt{y^2}=\pm\sqrt{4.5}$

Third - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{y=\pm 2.12}$. Therefore, the solution set is $\{-2.12,\ 2.12\}$ and the equation $2y^2=9$ can be factored to $\boldsymbol{(y-2.12)(y+2.12)=0}$.

Check: I. $Let\ y=-2.12\ in\quad 2y^2=9\ ;\ 2\cdot(-2.12)^2\overset{?}{=}9\ ;\ 2\cdot 4.5\overset{?}{=}9\ ;\ 9=9$

II. $Let\ y=-2.12\ in\quad 2y^2=9\ ;\ 2\cdot 2.12^2\overset{?}{=}9\ ;\ 2\cdot 4.5\overset{?}{=}9\ ;\ 9=9$

Practice Problems - Solving Quadratic Equations Using the Square Root Property Method

Section 1.3 Practice Problems - Solve the following equations using the Square Root Property method:

1. $(2y+5)^2=36$
2. $(x+1)^2=7$
3. $(2x-3)^2=1$
4. $x^2+3=0$
5. $(y-5)^2=5$
6. $16x^2-25=0$
7. $x^2-49=0$
8. $(3x-1)^2=25$
9. $(x-2)^2=-7$
10. $\left(x-\frac{1}{3}\right)^2=\frac{1}{9}$

1.4 Solving Quadratic Equations Using Completing-the-Square Method

One of the methods used in solving quadratic equations is called Completing-the-Square method. Note that this method involves construction of perfect square trinomials which was addressed in Section 3.5, Case I in the Mastering Algebra – Intermediate Level book. In this section we will learn how to solve quadratic equations of the form $ax^2 + bx + c = 0$, where $a = 1$ (case I) and where $a \rangle 1$ (Case II), using Completing-the-Square method.

Case I Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a = 1$, by Completing the Square

The following show the steps as to how quadratic equations, where the coefficient of the squared term is equal to one, are solved using Completing-the-Square method:

Step 1 Write the equation in the form of $x^2 + bx = -c$.

Step 2 a. Divide the coefficient of x by 2, i.e., $\frac{b}{2}$.

b. Square half the coefficient of x obtained in step 2a, i.e., $\left(\frac{b}{2}\right)^2$.

c. Add the square of half the coefficient of x to both sides of the equation, i.e.,

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2 .$$

d. Simplify the equation.

Step 3 Factor the trinomial on the left hand side of the equation as the square of a binomial, i.e., $\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$.

Step 4 Take the square root of both sides of the equation and solve for the x values, i.e.,

$$\sqrt{\left(x + \frac{b}{2}\right)^2} = \pm\sqrt{-c + \left(\frac{b}{2}\right)^2} \;;\; x + \frac{b}{2} = \pm\sqrt{-c + \left(\frac{b}{2}\right)^2} \;;\; x = -\frac{b}{2} \pm \sqrt{-c + \left(\frac{b}{2}\right)^2} .$$

Step 5 Check the answers by substituting the x values into the original equation.

Step 6 Write the quadratic equation in its factored form.

Examples with Steps

The following examples show the steps as to how quadratic equations, where the coefficient of the squared term is equal to one, are solved using Completing-the-Square method:

Example 1.4-1

Solve the quadratic equation $x^2 + 8x + 5 = 0$ by completing the square.

Solution:

Step 1 $\boxed{x^2 + 8x + 5 = 0}$; $\boxed{x^2 + 8x + 5 - 5 = -5}$; $\boxed{x^2 + 8x + 0 = -5}$; $\boxed{x^2 + 8x = -5}$

Step 2 $\boxed{x^2 + 8x = -5}$; $\boxed{x^2 + 8x + \left(\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2 = -5 + \left(\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2}$; $\boxed{x^2 + 8x + 4^2 = -5 + 4^2}$

; $\boxed{x^2+8x+16=-5+16}$; $\boxed{x^2+8x+16=11}$

Step 3 $\boxed{x^2+8x+16=11}$; $\boxed{(x+4)^2=11}$

Step 4 $\boxed{(x+4)^2=11}$; $\boxed{\sqrt{(x+4)^2}=\pm\sqrt{11}}$; $\boxed{x+4=\pm\sqrt{11}}$; $\boxed{x+4=\pm 3.3166}$ therefore:

I. $\boxed{x+4=+3.3166}$; $\boxed{x=3.3166-4}$; $\boxed{\boldsymbol{x=-0.6834}}$

II. $\boxed{x+4=-3.3166}$; $\boxed{x=-3.3166-4}$; $\boxed{\boldsymbol{x=-7.3166}}$

Thus, the solution set is $\{\mathbf{-7.3166, -0.6834}\}$.

Step 5 Check: Substitute $x=-0.6834$ and $x=-7.3166$ in $x^2+8x+5=0$

I. *Let* $x=-0.6834$ *in* $x^2+8x+5=0$; $(-0.6834)^2+(8\times-0.6834)+5\overset{?}{=}0$

; $0.467-5.467+5\overset{?}{=}0$; $-5+5\overset{?}{=}0$; $0=0$

II. *Let* $x=-7.3166$ *in* $x^2+8x+5=0$; $(-7.3166)^2+(8\times-7.3166)+5\overset{?}{=}0$

; $53.533-58.533+5\overset{?}{=}0$; $-5+5\overset{?}{=}0$; $0=0$

Step 6 Thus, the equation $x^2+8x+5=0$ can be factored to $(\boldsymbol{x+0.6834})(\boldsymbol{x+7.3166})=\mathbf{0}$

Example 1.4-2

Solve the quadratic equation $x^2-4x+3=0$ by completing the square.

Solution:

Step 1 $\boxed{x^2-4x+3=0}$; $\boxed{x^2-4x+3-3=-3}$; $\boxed{x^2-4x+0=-3}$; $\boxed{x^2-4x=-3}$

Step 2 $\boxed{x^2-4x=-3}$; $\boxed{x^2-4x+\left(-\frac{\cancel{4}^{\,2}}{\cancel{2}}\right)^2=-3+\left(-\frac{\cancel{4}^{\,2}}{\cancel{2}}\right)^2}$; $\boxed{x^2-4x+2^2=-3+2^2}$

; $\boxed{x^2-4x+4=-3+4}$; $\boxed{x^2-4x+4=1}$

Step 3 $\boxed{x^2-4x+4=1}$; $\boxed{(x-2)^2=1}$

Step 4 $\boxed{(x-2)^2=1}$; $\boxed{\sqrt{(x-2)^2}=\pm\sqrt{1}}$; $\boxed{x-2=\pm\sqrt{1}}$; $\boxed{x-2=\pm 1}$ therefore:

I. $\boxed{x-2=+1}$; $\boxed{x=2+1}$; $\boxed{\boldsymbol{x=3}}$

II. $\boxed{x-2=-1}$; $\boxed{x=2-1}$; $\boxed{\boldsymbol{x=1}}$

Thus, the solution set is $\{\mathbf{1, 3}\}$.

Step 5 Check: Substitute $x=3$ and $x=1$ in $x^2-4x+3=0$

I. *Let* $x = 3$ *in* $x^2 - 4x + 3 = 0$; $3^2 - (4 \times 3) + 3 \overset{?}{=} 0$; $9 - 12 + 3 \overset{?}{=} 0$; $12 - 12 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = 1$ *in* $x^2 - 4x + 3 = 0$; $1^2 - (4 \times 1) + 3 \overset{?}{=} 0$; $1 - 4 + 3 \overset{?}{=} 0$; $4 - 4 \overset{?}{=} 0$; $0 = 0$

Step 6 Thus, the equation $x^2 - 4x + 3 = 0$ can be factored to $\mathbf{(x-3)(x-1)=0}$

Example 1.4-3

Solve the quadratic equation $x^2 + x - 6 = 0$ by completing the square.

Solution:

Step 1 $\boxed{x^2 + x - 6 = 0}$; $\boxed{x^2 + x - 6 + 6 = +6}$; $\boxed{x^2 + x + 0 = 6}$; $\boxed{x^2 + x = 6}$

Step 2 $\boxed{x^2 + x = 6}$; $\boxed{x^2 + x + \left(\frac{1}{2}\right)^2 = 6 + \left(\frac{1}{2}\right)^2}$; $\boxed{x^2 + x + \frac{1}{4} = 6 + \frac{1}{4}}$; $\boxed{x^2 + x + \frac{1}{4} = \frac{6}{1} + \frac{1}{4}}$

; $\boxed{x^2 + x + \frac{1}{4} = \frac{(6 \cdot 4) + (1 \cdot 1)}{1 \cdot 4}}$; $\boxed{x^2 + x + \frac{1}{4} = \frac{24 + 1}{4}}$; $\boxed{x^2 + x + \frac{1}{4} = \frac{25}{4}}$

Step 3 $\boxed{x^2 + x + \frac{1}{4} = \frac{25}{4}}$; $\boxed{\left(x + \frac{1}{2}\right)^2 = \frac{25}{4}}$

Step 4 $\boxed{\left(x + \frac{1}{2}\right)^2 = \frac{25}{4}}$; $\boxed{\sqrt{\left(x + \frac{1}{2}\right)^2} = \pm\sqrt{\frac{25}{4}}}$; $\boxed{x + \frac{1}{2} = \pm\sqrt{\frac{5^2}{2^2}}}$; $\boxed{x + \frac{1}{2} = \pm\frac{5}{2}}$ therefore:

I. $\boxed{x + \frac{1}{2} = +\frac{5}{2}}$; $\boxed{x = -\frac{1}{2} + \frac{5}{2}}$; $\boxed{x = \frac{-1 + 5}{2}}$; $\boxed{x = \frac{\overset{2}{\cancel{4}}}{\cancel{2}}}$; $\boxed{\mathbf{x = 2}}$

II. $\boxed{x + \frac{1}{2} = -\frac{5}{2}}$; $\boxed{x = -\frac{1}{2} - \frac{5}{2}}$; $\boxed{x = \frac{-1 - 5}{2}}$; $\boxed{x = -\frac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{\mathbf{x = -3}}$

Thus, the solution set is $\{\mathbf{-3, 2}\}$.

Step 5 Check: Substitute $x = 2$ and $x = -3$ in $x^2 + x - 6 = 0$

I. *Let* $x = 2$ *in* $x^2 + x - 6 = 0$; $2^2 + 2 - 6 \overset{?}{=} 0$; $4 + 2 - 6 \overset{?}{=} 0$; $6 - 6 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = -3$ *in* $x^2 + x - 6 = 0$; $(-3)^2 - 3 - 6 \overset{?}{=} 0$; $9 - 3 - 6 \overset{?}{=} 0$; $9 - 9 \overset{?}{=} 0$; $0 = 0$

Step 6 Thus, the equation $x^2 + x - 6 = 0$ can be factored to $\mathbf{(x-2)(x+3)=0}$

Example 1.4-4

Solve the quadratic equation $x^2 - 6x + 2 = 0$ by completing the square.

Solution:

Step 1 $\boxed{x^2 - 6x + 2 = 0}$; $\boxed{x^2 - 6x + 2 - 2 = -2}$; $\boxed{x^2 - 6x + 0 = -2}$; $\boxed{x^2 - 6x = -2}$

Step 2 $\boxed{x^2 - 6x = -2}$; $\boxed{x^2 - 6x + \left(-\frac{\cancel{6}^{3}}{\cancel{2}}\right)^2 = -2 + \left(-\frac{\cancel{6}^{3}}{\cancel{2}}\right)^2}$; $\boxed{x^2 - 6x + 3^2 = -2 + 3^2}$

; $\boxed{x^2 - 6x + 9 = -2 + 9}$; $\boxed{x^2 - 6x + 9 = 7}$

Step 3 $\boxed{x^2 - 6x + 9 = 7}$; $\boxed{(x-3)^2 = 7}$

Step 4 $\boxed{(x-3)^2 = 7}$; $\boxed{\sqrt{(x-3)^2} = \pm\sqrt{7}}$; $\boxed{x - 3 = \pm\sqrt{7}}$; $\boxed{x - 3 = \pm 2.646}$ therefore:

I. $\boxed{x - 3 = +2.646}$; $\boxed{x = 3 + 2.646}$; $\boxed{\mathbf{x = 5.646}}$

II. $\boxed{x - 3 = -2.646}$; $\boxed{x = 3 - 2.646}$; $\boxed{\mathbf{x = 0.354}}$

Thus, the solution set is $\{\mathbf{0.354}, \mathbf{5.646}\}$.

Step 5 Check: Substitute $x = 5.646$ and $x = 0.354$ in $x^2 - 6x + 2 = 0$

I. *Let* $x = 5.646$ *in* $x^2 - 6x + 2 = 0$; $5.646^2 - (6 \times 5.646) + 2 \stackrel{?}{=} 0$

; $31.877 - 33.877 + 2 \stackrel{?}{=} 0$; $33.877 - 33.877 \stackrel{?}{=} 0$; $0 = 0$

II. *Let* $x = 0.354$ *in* $x^2 - 6x + 2 = 0$; $0.354^2 - (6 \times 0.354) + 2 \stackrel{?}{=} 0$; $0.13 - 2.13 + 2 \stackrel{?}{=} 0$

; $2.13 - 2.13 \stackrel{?}{=} 0$; $0 = 0$

Step 6 Thus, the equation $x^2 - 6x + 2 = 0$ can be factored to $(\mathbf{x - 5.646})(\mathbf{x - 0.354}) = \mathbf{0}$

Example 1.4-5

Solve the quadratic equation $x^2 + 2x + 5 = 0$ by completing the square.

Solution:

Step 1 $\boxed{x^2 + 2x + 5 = 0}$; $\boxed{x^2 + 2x + 5 - 5 = -5}$; $\boxed{x^2 + 2x + 0 = -5}$; $\boxed{x^2 + 2x = -5}$

Step 2 $\boxed{x^2 + 2x = -5}$; $\boxed{x^2 + 2x + \left(\frac{\cancel{2}}{\cancel{2}}\right)^2 = -5 + \left(\frac{\cancel{2}}{\cancel{2}}\right)^2}$; $\boxed{x^2 + 2x + 1^2 = -5 + 1^2}$

; $\boxed{x^2 + 2x + 1 = -5 + 1}$; $\boxed{x^2 + 2x + 1 = -4}$

Step 3 $\boxed{x^2 + 2x + 1 = -4}$; $\boxed{(x+1)^2 = -4}$

Step 4 $\boxed{(x+1)^2 = -4}$; $\boxed{\sqrt{(x+1)^2} = \pm\sqrt{-4}}$; $\boxed{x + 1 = \pm\sqrt{-4}}$ $\sqrt{-4}$ is not a real number.

Therefore, the equation $x^2 + 2x + 5 = 0$ **does not have any real solutions.**

Step 5 $\boxed{\textit{Not Applicable}}$

Step 6 $\boxed{\textit{Not Applicable}}$

Additional Examples - Solving Quadratic Equations of the Form $ax^2+bx+c=0$, where $a=1$, by Completing the Square

The following examples further illustrate how to solve quadratic equations using Completing the Square method:

Example 1.4-6

Solve the quadratic equation $x^2+3x-7=0$ using Completing-the-Square method.

Solution:

$$\boxed{x^2+3x-7=0}\;;\;\boxed{x^2+3x=7}\;;\;\boxed{x^2+3x+\left(\frac{3}{2}\right)^2=7+\left(\frac{3}{2}\right)^2}\;;\;\boxed{x^2+3x+\frac{9}{4}=7+\frac{9}{4}}\;;\;\boxed{\left(x+\frac{3}{2}\right)^2=\frac{7}{1}+\frac{9}{4}}$$

$$;\;\boxed{\left(x+\frac{3}{2}\right)^2=\frac{(7\cdot4)+(1\cdot9)}{1\cdot4}}\;;\;\boxed{\left(x+\frac{3}{2}\right)^2=\frac{28+9}{4}}\;;\;\boxed{\left(x+\frac{3}{2}\right)^2=\frac{37}{4}}\;;\;\boxed{\sqrt{\left(x+\frac{3}{2}\right)^2}=\pm\sqrt{\frac{37}{4}}}\;;\;\boxed{x+\frac{3}{2}=\pm\frac{\sqrt{37}}{2}}$$

therefore: I. $\boxed{x+\frac{3}{2}=+\frac{\sqrt{37}}{2}}\;;\;\boxed{x=\frac{\sqrt{37}}{2}-\frac{3}{2}}\;;\;\boxed{x=\frac{6.083-3}{2}}\;;\;\boxed{x=\frac{3.083}{2}}\;;\;\boxed{\mathbf{x=1.541}}$

II. $\boxed{x+\frac{3}{2}=-\frac{\sqrt{37}}{2}}\;;\;\boxed{x=-\frac{\sqrt{37}}{2}-\frac{3}{2}}\;;\;\boxed{x=\frac{-6.083-3}{2}}\;;\;\boxed{x=-\frac{9.083}{2}}\;;\;\boxed{\mathbf{x=-4.541}}$

and the solution set is $\{\mathbf{-4.541,\,1.541}\}$.

Check: I. $Let\ x=1.541\ in\quad x^2+3x-7=0\;;\;(1.541)^2+(3\times1.541)-7\overset{?}{=}0\;;\;2.38+4.62-7\overset{?}{=}0\;;\;0=0$

II. $Let\ x=-4.541\ in\quad x^2+3x-7=0\;;\;(-4.541)^2+(3\times-4.541)-7\overset{?}{=}0\;;\;20.62-13.62-7\overset{?}{=}0$

$;\;0=0$

Therefore, the equation $x^2+3x-7=0$ can be factored to $\mathbf{(x-1.541)(x+4.541)=0}$.

Example 1.4-7

Solve the quadratic equation $x^2-x-20=0$ using Completing-the-Square method.

Solution:

$$\boxed{x^2-x-20=0}\;;\;\boxed{x^2-x=20}\;;\;\boxed{x^2-x+\left(-\frac{1}{2}\right)^2=20+\left(-\frac{1}{2}\right)^2}\;;\;\boxed{x^2-x+\frac{1}{4}=20+\frac{1}{4}}\;;\;\boxed{\left(x-\frac{1}{2}\right)^2=\frac{20}{1}+\frac{1}{4}}$$

$$;\;\boxed{\left(x-\frac{1}{2}\right)^2=\frac{(20\cdot4)+(1\cdot1)}{1\cdot4}}\;;\;\boxed{\left(x-\frac{1}{2}\right)^2=\frac{80+1}{4}}\;;\;\boxed{\left(x-\frac{1}{2}\right)^2=\frac{81}{4}}\;;\;\boxed{\sqrt{\left(x-\frac{1}{2}\right)^2}=\pm\sqrt{\frac{81}{4}}}\;;\;\boxed{x-\frac{1}{2}=\pm\frac{9}{2}}$$

therefore: I. $\boxed{x-\frac{1}{2}=+\frac{9}{2}}\;;\;\boxed{x=\frac{9}{2}+\frac{1}{2}}\;;\;\boxed{x=\frac{9+1}{2}}\;;\;\boxed{x=\frac{\overset{5}{\cancel{10}}}{\cancel{2}}}\;;\;\boxed{\mathbf{x=5}}$

II. $\boxed{x-\frac{1}{2}=-\frac{9}{2}}\;;\;\boxed{x=-\frac{9}{2}+\frac{1}{2}}\;;\;\boxed{x=\frac{-9+1}{2}}\;;\;\boxed{x=-\frac{\overset{4}{\cancel{8}}}{\cancel{2}}}\;;\;\boxed{\mathbf{x=-4}}$

and the solution set is $\{\mathbf{-4,\,5}\}$.

Check: I. $\text{Let } x = 5 \text{ in } \quad x^2 - x - 20 = 0 \; ; \; 5^2 - 5 - 20 \stackrel{?}{=} 0 \; ; \; 25 - 25 \stackrel{?}{=} 0 \; ; \; 0 = 0$

II. $\text{Let } x = -4 \text{ in } \quad x^2 - x - 20 = 0 \; ; \; (-4)^2 - (-4) - 20 \stackrel{?}{=} 0 \; ; \; 16 + 4 - 20 \stackrel{?}{=} 0 \; ; \; 20 - 20 \stackrel{?}{=} 0 \; ; \; 0 = 0$

Therefore, the equation $x^2 - x - 20 = 0$ can be factored to $\mathbf{(x-5)(x+4)=0}$.

Example 1.4-8

Solve the quadratic equation $x^2 + 5x + 6 = 0$ using Completing-the-Square method.

Solution:

$$\boxed{x^2 + 5x + 6 = 0} \; ; \; \boxed{x^2 + 5x = -6} \; ; \; \boxed{x^2 + 5x + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2} \; ; \; \boxed{x^2 + 5x + \frac{25}{4} = -6 + \frac{25}{4}} \; ; \; \boxed{\left(x + \frac{5}{2}\right)^2 = -\frac{6}{1} + \frac{25}{4}}$$

$$; \; \boxed{\left(x + \frac{5}{2}\right)^2 = \frac{(-6 \cdot 4) + (1 \cdot 25)}{1 \cdot 4}} \; ; \; \boxed{\left(x + \frac{5}{2}\right)^2 = \frac{-24 + 25}{4}} \; ; \; \boxed{\left(x + \frac{5}{2}\right)^2 = \frac{1}{4}} \; ; \; \boxed{\sqrt{\left(x + \frac{5}{2}\right)^2} = \pm\sqrt{\frac{1}{4}}} \; ; \; \boxed{x + \frac{5}{2} = \pm\frac{1}{2}}$$

therefore: I. $\boxed{x + \frac{5}{2} = +\frac{1}{2}} \; ; \; \boxed{x = \frac{1}{2} - \frac{5}{2}} \; ; \; \boxed{x = \frac{1-5}{2}} \; ; \; \boxed{x = -\frac{\overset{2}{\cancel{4}}}{\cancel{2}}} \; ; \; \boxed{x = -\frac{2}{1}} \; ; \; \boxed{\mathbf{x = -2}}$

II. $\boxed{x + \frac{5}{2} = -\frac{1}{2}} \; ; \; \boxed{x = -\frac{1}{2} - \frac{5}{2}} \; ; \; \boxed{x = \frac{-1-5}{2}} \; ; \; \boxed{x = -\frac{\overset{3}{\cancel{6}}}{\cancel{2}}} \; ; \; \boxed{x = -\frac{3}{1}} \; ; \; \boxed{\mathbf{x = -3}}$

and the solution set is $\{\mathbf{-3, -2}\}$.

Check: I. $\text{Let } x = -2 \text{ in } \quad x^2 + 5x + 6 = 0 \; ; \; (-2)^2 + (5 \times -2) + 6 \stackrel{?}{=} 0 \; ; \; 4 - 10 + 6 \stackrel{?}{=} 0 \; ; \; 10 - 10 \stackrel{?}{=} 0 \; ; \; 0 = 0$

II. $\text{Let } x = -3 \text{ in } \quad x^2 + 5x + 6 = 0 \; ; \; (-3)^2 + (5 \times -3) + 6 \stackrel{?}{=} 0 \; ; \; 9 - 15 + 6 \stackrel{?}{=} 0 \; ; \; 15 - 15 \stackrel{?}{=} 0 \; ; \; 0 = 0$

Therefore, the equation $x^2 + 5x + 6 = 0$ can be factored to $\mathbf{(x+2)(x+3)=0}$.

Example 1.4-9

Solve the quadratic equation $y^2 - 9y + 11 = 0$ using Completing-the-Square method.

Solution:

$$\boxed{y^2 - 9y + 11 = 0} \; ; \; \boxed{y^2 - 9y = -11} \; ; \; \boxed{y^2 - 9y + \left(-\frac{9}{2}\right)^2 = -11 + \left(-\frac{9}{2}\right)^2} \; ; \; \boxed{y^2 - 9y + \frac{81}{4} = -11 + \frac{81}{4}}$$

$$; \; \boxed{\left(y - \frac{9}{2}\right)^2 = -\frac{11}{1} + \frac{81}{4}} \; ; \; \boxed{\left(y - \frac{9}{2}\right)^2 = \frac{(-11 \cdot 4) + (1 \cdot 81)}{1 \cdot 4}} \; ; \; \boxed{\left(y - \frac{9}{2}\right)^2 = \frac{-44 + 81}{4}} \; ; \; \boxed{\left(y - \frac{9}{2}\right)^2 = \frac{37}{4}}$$

$; \; \boxed{\sqrt{\left(y - \frac{9}{2}\right)^2} = \pm\sqrt{\frac{37}{4}}} \; ; \; \boxed{y - \frac{9}{2} = \pm\frac{\sqrt{37}}{2}}$ therefore:

I. $\boxed{y - \frac{9}{2} = +\frac{\sqrt{37}}{2}} \; ; \; \boxed{y = \frac{\sqrt{37}}{2} + \frac{9}{2}} \; ; \; \boxed{y = \frac{6.083 + 9}{2}} \; ; \; \boxed{y = \frac{15.083}{2}} \; ; \; \boxed{\mathbf{y = 7.541}}$

II. $\boxed{y - \frac{9}{2} = -\frac{\sqrt{37}}{2}} \; ; \; \boxed{y = -\frac{\sqrt{37}}{2} + \frac{9}{2}} \; ; \; \boxed{y = \frac{-6.083 + 9}{2}} \; ; \; \boxed{y = \frac{2.917}{2}} \; ; \; \boxed{\mathbf{y = 1.459}}$

and the solution set is $\{\mathbf{1.459}, \mathbf{7.541}\}$.

Check: I. $Let\ y = 7.541\ \ in\ \ y^2 - 9y + 11 = 0$; $(7.541)^2 + (-9 \times 7.541) + 11 \stackrel{?}{=} 0$; $56.87 - 67.87 + 11 \stackrel{?}{=} 0$; $67.87 - 67.87 \stackrel{?}{=} 0$; $0 = 0$

II. $Let\ y = 1.459\ \ in\ \ y^2 - 9y + 11 = 0$; $(1.459)^2 + (-9 \times 1.459) + 11 \stackrel{?}{=} 0$; $2.13 - 13.13 + 11 \stackrel{?}{=} 0$; $13.13 - 13.13 \stackrel{?}{=} 0$; $0 = 0$

Therefore, the equation $y^2 - 9y + 11 = 0$ can be factored to $\mathbf{(y - 7.541)(y - 1.459) = 0}$.

Example 1.4-10

Solve the quadratic equation $x(x-2) - 24 = 0$ using Completing-the-Square method.

Solution:

$\boxed{x(x-2) - 24 = 0}$; $\boxed{x^2 - 2x - 24 = 0}$; $\boxed{x^2 - 2x = 24}$; $\boxed{x^2 - 2x + \left(\frac{2}{2}\right)^2 = 24 + \left(\frac{2}{2}\right)^2}$; $\boxed{x^2 - 2x + 1 = 24 + 1}$

; $\boxed{(x-1)^2 = 25}$; $\boxed{\sqrt{(x-1)^2} = \pm\sqrt{25}}$; $\boxed{x - 1 = \pm 5}$ therefore:

I. $\boxed{x - 1 = +5}$; $\boxed{x = 5 + 1}$; $\boxed{\mathbf{x = 6}}$

II. $\boxed{x - 1 = -5}$; $\boxed{x = -5 + 1}$; $\boxed{\mathbf{x = -4}}$

and the solution set is $\{\mathbf{-4}, \mathbf{6}\}$.

Check: I. $Let\ x = 6\ \ in\ \ x(x-2) - 24 = 0$; $6 \cdot (6-2) - 24 \stackrel{?}{=} 0$; $6 \cdot 4 - 24 \stackrel{?}{=} 0$; $24 - 24 \stackrel{?}{=} 0$; $0 = 0$

II. $Let\ x = -4\ \ in\ \ x(x-2) - 24 = 0$; $-4 \cdot (-4-2) - 24 \stackrel{?}{=} 0$; $-4 \cdot (-6) - 24 \stackrel{?}{=} 0$; $24 - 24 \stackrel{?}{=} 0$; $0 = 0$

Therefore, the equation $x(x-2) - 24 = 0$ can be factored to $\mathbf{(x - 6)(x + 4) = 0}$.

Practice Problems - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a = 1$, by Completing the Square

Section 1.4 Case I Practice Problems - Solve the following quadratic equations using Completing-the-Square method:

1. $x^2 + 10x - 2 = 0$
2. $x^2 - x - 1 = 0$
3. $x(x+2) = 80$
4. $y^2 - 10y + 5 = 0$
5. $x^2 + 4x - 5 = 0$
6. $y^2 + 4y = 14$
7. $w^2 + \frac{1}{3}w - \frac{1}{2} = 0$
8. $z^2 + 3z = -\frac{1}{4}$
9. $z^2 + \frac{5}{3}z - \frac{1}{2} = 0$
10. $x^2 - 6x = -4$

Case II Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a \rangle 1$, by Completing the Square

The following show the steps as to how quadratic equations, where the coefficient of the squared term is not equal to one, are solved using Completing-the-Square method:

Step 1 Write the equation in the form of $ax^2 + bx = -c$.

Step 2 Divide both sides of the equation by a, i.e., $\frac{\cancel{a}x^2}{\cancel{a}} + \frac{bx}{a} = -\frac{c}{a}$; $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

Step 3 a. Divide the coefficient of x by 2, i.e., $\frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$.

b. Square half the coefficient of x obtained in step 3a, i.e., $\left(\frac{b}{2a}\right)^2$

c. Add the square of half the coefficient of x to both sides of the equation, i.e.,

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 .$$

d. Simplify the equation.

Step 4 Factor the trinomial on the left hand side of the equation as the square of a binomial, i.e., $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$.

Step 5 Take the square root of both sides of the equation and solve for the x values, i.e.,

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2} \;;\; x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2} \;;\; x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2} .$$

Step 6 Check the answers by substituting the x values into the original equation.

Step 7 Write the quadratic equation in its factored form.

Examples with Steps

The following examples show the steps as to how quadratic equations, where the coefficient of the squared term is not equal to one, are solved using completing-the-square method:

Example 1.4-11

Solve the quadratic equation $3x^2 - 16x + 5 = 0$ by completing the square.

Solution:

Step 1 $\boxed{3x^2 - 16x + 5 = 0}$; $\boxed{3x^2 - 16x + 5 - 5 = -5}$; $\boxed{3x^2 - 16x + 0 = -5}$; $\boxed{3x^2 - 16x = -5}$

Step 2 $\boxed{3x^2 - 16x = -5}$; $\boxed{\frac{\cancel{3}}{\cancel{3}}x^2 - \frac{16}{3}x = -\frac{5}{3}}$; $\boxed{x^2 - \frac{16}{3}x = -\frac{5}{3}}$

Step 3 $\boxed{x^2 - \frac{16}{3}x = -\frac{5}{3}}$; $\boxed{x^2 - \frac{16}{3}x + \left(-\frac{\cancel{16}^{\,8}}{\cancel{6}_{\,3}}\right)^2 = -\frac{5}{3} + \left(-\frac{\cancel{16}^{\,8}}{\cancel{6}_{\,3}}\right)^2}$; $\boxed{x^2 - \frac{16}{3}x + \left(-\frac{8}{3}\right)^2 = -\frac{5}{3} + \left(-\frac{8}{3}\right)^2}$

; $\boxed{x^2 - \frac{16}{3}x + \frac{64}{9} = -\frac{5}{3} + \frac{64}{9}}$; $\boxed{x^2 - \frac{16}{3}x + \frac{64}{9} = \frac{(-5 \cdot 9) + (3 \cdot 64)}{3 \cdot 9}}$

; $\boxed{x^2-\frac{16}{3}x+\frac{64}{9}=\frac{-45+192}{27}}$; $\boxed{x^2-\frac{16}{3}x+\frac{64}{9}=\frac{\cancel{147}^{49}}{\cancel{27}_{9}}}$; $\boxed{x^2-\frac{16}{3}x+\frac{64}{9}=\frac{49}{9}}$

Step 4 $\boxed{x^2-\frac{16}{3}x+\frac{64}{9}=\frac{49}{9}}$; $\boxed{\left(x-\frac{8}{3}\right)^2=\frac{49}{9}}$

Step 5 $\boxed{\left(x-\frac{8}{3}\right)^2=\frac{49}{9}}$; $\boxed{\sqrt{\left(x-\frac{8}{3}\right)^2}=\pm\sqrt{\frac{49}{9}}}$; $\boxed{x-\frac{8}{3}=\pm\sqrt{\frac{7^2}{3^2}}}$; $\boxed{x-\frac{8}{3}=\pm\frac{7}{3}}$ therefore:

I. $\boxed{x-\frac{8}{3}=+\frac{7}{3}}$; $\boxed{x=\frac{8}{3}+\frac{7}{3}}$; $\boxed{x=\frac{8+7}{3}}$; $\boxed{x=\frac{\cancel{15}^{5}}{\cancel{3}}}$; $\boxed{x=\frac{5}{1}}$; $\boxed{\boldsymbol{x=5}}$

II. $\boxed{x-\frac{8}{3}=-\frac{7}{3}}$; $\boxed{x=\frac{8}{3}-\frac{7}{3}}$; $\boxed{x=\frac{8-7}{3}}$; $\boxed{\boldsymbol{x=\frac{1}{3}}}$

and the solution set is $\left\{\boldsymbol{\frac{1}{3}, 5}\right\}$.

Step 6 Check: Substitute $x=5$ and $x=\frac{1}{3}$ in $3x^2-16x+5=0$

I. *Let* $x=5$ *in* $3x^2-16x+5=0$; $3\cdot5^2-(16\times5)+5\overset{?}{=}0$; $75-80+5\overset{?}{=}0$

; $80-80+5\overset{?}{=}0$; $0=0$

II. *Let* $x=\frac{1}{3}$ *in* $3x^2-16x+5=0$; $3\cdot\left(\frac{1}{3}\right)^2-\left(16\times\frac{1}{3}\right)+5\overset{?}{=}0$; $\frac{\cancel{3}}{\cancel{9}_{3}}-\frac{16}{3}+5\overset{?}{=}0$

; $\frac{1}{3}-\frac{16}{3}+5\overset{?}{=}0$; $\frac{1-16}{3}+5\overset{?}{=}0$; $-\frac{\cancel{15}^{5}}{\cancel{3}}+5\overset{?}{=}0$; $-5+5\overset{?}{=}0$; $0=0$

Step 7 Thus, the equation $3x^2-16x+5=0$ can be factored to $(x-5)\left(x-\frac{1}{3}\right)=0$ which is the same as $\boldsymbol{(x-5)(3x-1)=0}$

Example 1.4-12

Solve the quadratic equation $2x^2+3x-6=0$ by completing the square.

Solution:

Step 1 $\boxed{2x^2+3x-6=0}$; $\boxed{2x^2+3x-6+6=+6}$; $\boxed{2x^2+3x+0=+6}$; $\boxed{2x^2+3x=6}$

Step 2 $\boxed{2x^2+3x=6}$; $\boxed{\frac{\cancel{2}}{\cancel{2}}x^2+\frac{3}{2}x=\frac{\cancel{6}^{3}}{\cancel{2}}}$; $\boxed{x^2+\frac{3}{2}x=\frac{3}{1}}$; $\boxed{x^2+\frac{3}{2}x=3}$

Step 3 $\boxed{x^2+\frac{3}{2}x=3}$; $\boxed{x^2+\frac{3}{2}x+\left(\frac{3}{4}\right)^2=3+\left(\frac{3}{4}\right)^2}$; $\boxed{x^2+\frac{3}{2}x+\frac{9}{16}=3+\frac{9}{16}}$

; $\boxed{x^2+\frac{3}{2}x+\frac{9}{16}=\frac{3}{1}+\frac{9}{16}}$; $\boxed{x^2+\frac{3}{2}x+\frac{9}{16}=\frac{48+9}{16}}$; $\boxed{x^2+\frac{3}{2}x+\frac{9}{16}=\frac{57}{16}}$

Step 4 $\boxed{x^2+\frac{3}{2}x+\frac{9}{16}=\frac{57}{16}}$; $\boxed{\left(x+\frac{3}{4}\right)^2=\frac{57}{16}}$

Step 5 $\boxed{\left(x+\frac{3}{4}\right)^2=\frac{57}{16}}$; $\boxed{\sqrt{\left(x+\frac{3}{4}\right)^2}=\pm\sqrt{\frac{57}{16}}}$; $\boxed{x+\frac{3}{4}=\pm\sqrt{\frac{57}{4^2}}}$; $\boxed{x+\frac{3}{4}=\pm\frac{\sqrt{57}}{4}}$ therefore:

I. $\boxed{x+\frac{3}{4}=+\frac{\sqrt{57}}{4}}$; $\boxed{x=\frac{7.55}{4}-\frac{3}{4}}$; $\boxed{x=\frac{7.55-3}{4}}$; $\boxed{x=\frac{4.55}{4}}$; $\boxed{\mathbf{x=1.138}}$

II. $\boxed{x+\frac{3}{4}=-\frac{\sqrt{57}}{4}}$; $\boxed{x=-\frac{7.55}{4}-\frac{3}{4}}$; $\boxed{x=\frac{-7.55-3}{4}}$; $\boxed{x=\frac{-10.55}{4}}$; $\boxed{\mathbf{x=-2.638}}$

and the solution set is $\{\mathbf{-2.638, 1.138}\}$.

Step 6 Check: Substitute $x=1.138$ and $x=-2.638$ in $2x^2+3x-6=0$

I. *Let* $x=1.138$ *in* $2x^2+3x-6=0$; $2\cdot(1.138)^2+(3\times 1.138)-6\overset{?}{=}0$

; $2.59+3.41-6\overset{?}{=}0$; $6-6\overset{?}{=}0$; $0=0$

II. *Let* $x=-2.638$ *in* $2x^2+3x-6=0$; $2\cdot(-2.638)^2+(3\times -2.638)-6\overset{?}{=}0$

; $13.92-7.92-6\overset{?}{=}0$; $13.92-13.92\overset{?}{=}0$; $0=0$

Step 7 Thus, the equation $2x^2+3x-6=0$ which is equal to $x^2+1.5x-3=0$ can be factored to $\mathbf{(x-1.138)(x+2.638)=0}$

Example 1.4-13

Solve the quadratic equation $8x^2+8x-30=0$ by completing the square.

Solution:

Step 1 $\boxed{8x^2+8x-30=0}$; $\boxed{8x^2+8x-30+30=+30}$; $\boxed{8x^2+8x+0=30}$; $\boxed{8x^2+8x=30}$

Step 2 $\boxed{8x^2+8x=30}$; $\boxed{\frac{\cancel{8}}{\cancel{8}}x^2+\frac{\cancel{8}}{\cancel{8}}x=\frac{\overset{15}{\cancel{30}}}{\underset{4}{\cancel{8}}}}$; $\boxed{x^2+x=\frac{15}{4}}$

Step 3 $\boxed{x^2+x=\frac{15}{4}}$; $\boxed{x^2+x+\left(\frac{1}{2}\right)^2=\frac{15}{4}+\left(\frac{1}{2}\right)^2}$; $\boxed{x^2+x+\frac{1}{4}=\frac{15}{4}+\frac{1}{4}}$; $\boxed{x^2+x+\frac{1}{4}=\frac{15+1}{4}}$

; $\boxed{x^2+x+\frac{1}{4}=\frac{\overset{4}{\cancel{16}}}{\cancel{4}}}$; $\boxed{x^2+x+\frac{1}{4}=4}$

Step 4 $\boxed{x^2+x+\frac{1}{4}=4}$; $\boxed{\left(x+\frac{1}{2}\right)^2=4}$

Step 5 $\boxed{\left(x+\frac{1}{2}\right)^2=4}$; $\boxed{\sqrt{\left(x+\frac{1}{2}\right)^2}=\pm\sqrt{4}}$; $\boxed{x+\frac{1}{2}=\pm 2}$ therefore:

I. $\boxed{x+\frac{1}{2}=+2}$; $\boxed{x=-\frac{1}{2}+\frac{2}{1}}$; $\boxed{x=\frac{-(1\cdot 1)+(2\cdot 2)}{1\cdot 2}}$; $\boxed{x=\frac{-1+4}{2}}$; $\boxed{\boldsymbol{x=\frac{3}{2}}}$

II. $\boxed{x+\frac{1}{2}=-2}$; $\boxed{x=-\frac{1}{2}-\frac{2}{1}}$; $\boxed{x=\frac{-(1\cdot 1)-(2\cdot 2)}{1\cdot 2}}$; $\boxed{x=\frac{-1-4}{2}}$; $\boxed{\boldsymbol{x=-\frac{5}{2}}}$

and the solution set is $\left\{-\frac{\mathbf{5}}{\mathbf{2}},\frac{\mathbf{3}}{\mathbf{2}}\right\}$.

Step 6 Check: Substitute $x=\frac{3}{2}$ and $x=-\frac{5}{2}$ in $8x^2+8x-30=0$

I. *Let* $x=\frac{3}{2}$ *in* $8x^2+8x-30=0$; $8\cdot\left(\frac{3}{2}\right)^2+8\cdot\frac{3}{2}-30\stackrel{?}{=}0$; $8\cdot\frac{9}{4}+\frac{24}{2}-30\stackrel{?}{=}0$

; $\frac{\overset{18}{\cancel{72}}}{\cancel{4}}+\frac{\overset{12}{\cancel{24}}}{\cancel{2}}-30\stackrel{?}{=}0$; $18+12-30\stackrel{?}{=}0$; $30-30\stackrel{?}{=}0$; $0=0$

II. *Let* $x=-\frac{5}{2}$ *in* $8x^2+8x-30=0$; $8\cdot\left(-\frac{5}{2}\right)^2+8\cdot\left(-\frac{5}{2}\right)-30\stackrel{?}{=}0$

; $8\cdot\frac{25}{4}-\frac{40}{2}-30\stackrel{?}{=}0$; $\frac{\overset{50}{\cancel{200}}}{\cancel{4}}-\frac{\overset{20}{\cancel{40}}}{\cancel{2}}-30\stackrel{?}{=}0$; $50-20-30\stackrel{?}{=}0$; $50-50\stackrel{?}{=}0$; $0=0$

Step 7 Thus, the equation $8x^2+8x-30=0$ which is the same as $4x^2+4x-15=0$ can be factored to $\left(x-\frac{3}{2}\right)\left(x+\frac{5}{2}\right)=0$ which is the same as $\boldsymbol{(2x-3)(2x+5)=0}$

Example 1.4-14

Solve the quadratic equation $2u^2+6u-7=0$ by completing the square.

Solution:

Step 1 $\boxed{2u^2+6u-7=0}$; $\boxed{2u^2+6u-7+7=+7}$; $\boxed{2u^2+6u+0=7}$; $\boxed{2u^2+6u=7}$

Step 2 $\boxed{2u^2+6u=7}$; $\boxed{\frac{\cancel{2}}{\cancel{2}}u^2+\frac{\overset{3}{\cancel{6}}}{\cancel{2}}u=\frac{7}{2}}$; $\boxed{u^2+3u=\frac{7}{2}}$

Step 3 $\boxed{u^2+3u=\frac{7}{2}}$; $\boxed{u^2+3u+\left(\frac{3}{2}\right)^2=\frac{7}{2}+\left(\frac{3}{2}\right)^2}$; $\boxed{u^2+3u+\frac{9}{4}=\frac{7}{2}+\frac{9}{4}}$

; $\boxed{u^2+3u+\frac{9}{4}=\frac{(7\cdot 4)+(2\cdot 9)}{2\cdot 4}}$; $\boxed{u^2+3u+\frac{9}{4}=\frac{28+18}{8}}$; $\boxed{u^2+3u+\frac{9}{4}=\frac{46}{8}}$

Step 4 $\boxed{u^2+3u+\frac{9}{4}=\frac{46}{8}}$; $\boxed{\left(u+\frac{3}{2}\right)^2=\frac{\overset{23}{\cancel{46}}}{\underset{4}{\cancel{8}}}}$; $\boxed{\left(u+\frac{3}{2}\right)^2=\frac{23}{4}}$

Step 5 $\boxed{\left(u+\frac{3}{2}\right)^2=\frac{23}{4}}$; $\boxed{\sqrt{\left(u+\frac{3}{2}\right)^2}=\pm\sqrt{\frac{23}{4}}}$; $\boxed{u+\frac{3}{2}=\pm\sqrt{\frac{23}{2^2}}}$; $\boxed{u+\frac{3}{2}=\pm\frac{\sqrt{23}}{2}}$ therefore:

I. $\boxed{u+\frac{3}{2}=+\frac{\sqrt{23}}{2}}$; $\boxed{u=-\frac{3}{2}+\frac{\sqrt{23}}{2}}$; $\boxed{u=\frac{-3+\sqrt{23}}{2}}$; $\boxed{u=\frac{-3+4.8}{2}}$; $\boxed{u=\frac{1.8}{2}}$; $\boxed{\boldsymbol{u=0.9}}$

II. $\boxed{u+\frac{3}{2}=-\frac{\sqrt{23}}{2}}$; $\boxed{u=-\frac{3}{2}-\frac{\sqrt{23}}{2}}$; $\boxed{u=\frac{-3-\sqrt{23}}{2}}$; $\boxed{u=\frac{-3-4.8}{2}}$; $\boxed{u=\frac{-7.8}{2}}$; $\boxed{\boldsymbol{u=-3.9}}$

and the solution set is $\{\mathbf{-3.9,\ 0.9}\}$.

Step 6 Check: Substitute $u=0.9$ and $u=-3.9$ in $2u^2+6u-7=0$

I. *Let* $u=0.9$ *in* $2u^2+6u-7=0$; $2\cdot 0.9^2+(6\times 0.9)-7\overset{?}{=}0$; $1.6+5.4-7\overset{?}{=}0$; $7-7\overset{?}{=}0$; $0=0$

II. *Let* $u=-3.9$ *in* $2u^2+6u-7=0$; $2\cdot(-3.9)^2+(6\times -3.9)-7\overset{?}{=}0$; $2\times 15.2-23.4-7\overset{?}{=}0$; $30.4-23.4-7\overset{?}{=}0$; $7-7\overset{?}{=}0$; $0=0$

Step 7 Thus, the equation $2u^2+6u-7=0$, which is the same as $u^2+3u-3.5=0$, can be factored to $\boldsymbol{(u-0.9)(u+3.9)=0}$.

Example 1.4-15

Solve the quadratic equation $4a^2+24a-5=0$ by completing the square.

Solution:

Step 1 $\boxed{4a^2+24a-5=0}$; $\boxed{4a^2+24a-5+5=+5}$; $\boxed{4a^2+24a+0=5}$; $\boxed{4a^2+24a=5}$

Step 2 $\boxed{4a^2+24a=5}$; $\boxed{\frac{\cancel{4}}{\cancel{4}}a^2+\frac{\overset{6}{\cancel{24}}}{\cancel{4}}a=\frac{5}{4}}$; $\boxed{a^2+6a=\frac{5}{4}}$

Step 3 $\boxed{a^2+6a=\frac{5}{4}}$; $\boxed{a^2+6a+\left(\frac{\overset{3}{\cancel{6}}}{\cancel{2}}\right)^2=\frac{5}{4}+\left(\frac{\overset{3}{\cancel{6}}}{\cancel{2}}\right)^2}$; $\boxed{a^2+6a+\left(\frac{3}{1}\right)^2=\frac{5}{4}+\left(\frac{3}{1}\right)^2}$

; $\boxed{a^2+6a+3^2=\frac{5}{4}+3^2}$; $\boxed{a^2+6a+9=\frac{5}{4}+\frac{9}{1}}$; $\boxed{a^2+6a+9=\frac{(5\cdot 1)+(9\cdot 4)}{4\cdot 1}}$

; $\boxed{a^2+6a+9=\frac{5+36}{4}}$; $\boxed{a^2+6a+9=\frac{41}{4}}$

Step 4 $\boxed{a^2+6a+9=\frac{41}{4}}$; $\boxed{(a+3)^2=10.25}$

Step 5 $\boxed{(a+3)^2=10.25}$; $\boxed{\sqrt{(a+3)^2}=\pm\sqrt{10.25}}$; $\boxed{a+3=\pm 3.2}$ therefore:

I. $\boxed{a+3=+3.2}$; $\boxed{a=3.2-3}$; $\boxed{\boldsymbol{a=0.2}}$

II. $\boxed{a+3=-3.2}$; $\boxed{a=-3.2-3}$; $\boxed{\boldsymbol{a=-6.2}}$

and the solution set is $\{\mathbf{-6.2, 0.2}\}$.

Step 6 Check: Substitute $a = 0.2$ and $a = -6.2$ in $4a^2 + 24a - 5 = 0$

I. *Let* $a = 0.2$ *in* $4a^2 + 24a - 5 = 0$; $4 \cdot 0.2^2 + (24 \times 0.2) - 5 \overset{?}{=} 0$; $4 \times 0.04 + 4.8 - 5 \overset{?}{=} 0$

; $0.16 + 4.8 - 5 \overset{?}{=} 0$; $5 - 5 \overset{?}{=} 0$; $0 = 0$

II. *Let* $a = -6.2$ *in* $4a^2 + 24a - 5 = 0$; $4 \cdot (-6.2)^2 + (24 \times -6.2) - 5 \overset{?}{=} 0$;

; $4 \times 38.44 - 148.8 - 5 \overset{?}{=} 0$; $153.8 - 148.8 - 5 \overset{?}{=} 0$; $153.8 - 153.8 \overset{?}{=} 0$; $0 = 0$

Step 7 Thus, the equation $4a^2 + 24a - 5 = 0$ can be factored to $\mathbf{(a - 0.2)(a + 6.2) = 0}$.

Additional Examples - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a \rangle 1$, by Completing the Square

The following examples further illustrate how to solve quadratic equations, where the coefficient of the squared term is not equal to one, using Completing-the-Square method:

Example 1.4-16

Solve the quadratic equation $3x^2 + 2x - 1 = 0$ using Completing-the-Square method.

Solution:

$\boxed{3x^2 + 2x - 1 = 0}$; $\boxed{3x^2 + 2x = 1}$; $\boxed{\frac{\cancel{3}}{\cancel{3}}x^2 + \frac{2}{3}x = \frac{1}{3}}$; $\boxed{x^2 + \frac{2}{3}x = \frac{1}{3}}$; $\boxed{x^2 + \frac{2}{3}x + \left(\frac{\cancel{2}}{\cancel{6}}\right)^2 = \frac{1}{3} + \left(\frac{\cancel{2}}{\cancel{6}}\right)^2}$

; $\boxed{x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \frac{1}{3} + \left(\frac{1}{3}\right)^2}$; $\boxed{x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}}$; $\boxed{\left(x + \frac{1}{3}\right)^2 = \frac{1}{3} + \frac{1}{9}}$; $\boxed{\left(x + \frac{1}{3}\right)^2 = \frac{(1 \cdot 9) + (1 \cdot 3)}{3 \cdot 9}}$

; $\boxed{\left(x + \frac{1}{3}\right)^2 = \frac{9 + 3}{27}}$; $\boxed{\left(x + \frac{1}{3}\right)^2 = \frac{12}{27}}$; $\boxed{\sqrt{\left(x + \frac{1}{3}\right)^2} = \pm\sqrt{\frac{12}{27}}}$; $\boxed{x + \frac{1}{3} = \pm\sqrt{\frac{12}{27}}}$; $\boxed{x + \frac{1}{3} = \pm\sqrt{0.44}}$

; $\boxed{x + 0.33 = \pm 0.67}$ therefore:

I. $\boxed{x + 0.33 = +0.67}$; $\boxed{x = 0.67 - 0.33}$; $\boxed{\mathbf{x = 0.34}}$ II. $\boxed{x + 0.33 = -0.67}$; $\boxed{x = -0.67 - 0.33}$; $\boxed{\mathbf{x = -1}}$

and the solution set is $\{\mathbf{-1, 0.34}\}$.

Check: I. *Let* $x = -1$ *in* $3x^2 + 2x - 1 = 0$; $3 \cdot (-1)^2 + (2 \cdot -1) - 1 \overset{?}{=} 0$; $3 \cdot 1 - 2 - 1 \overset{?}{=} 0$; $3 - 3 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = 0.34$ *in* $3x^2 + 2x - 1 = 0$; $3 \cdot 0.34^2 + (2 \cdot 0.34) - 1 \overset{?}{=} 0$; $3 \cdot 0.11 + 0.68 - 1 \overset{?}{=} 0$

; $0.33 + 0.68 - 1 \overset{?}{=} 0$; $1 - 1 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $3x^2 + 2x - 1 = 0$ can be factored to $\mathbf{(x - 0.34)(x + 1) = 0}$.

Example 1.4-17

Solve the quadratic equation $3y^2 - 8y + 2 = 0$ using Completing-the-Square method.

Solution:

$\boxed{3y^2 - 8y + 2 = 0}$; $\boxed{3y^2 - 8y = -2}$; $\boxed{\frac{\cancel{3}}{\cancel{3}}y^2 - \frac{8}{3}y = -\frac{2}{3}}$; $\boxed{y^2 - \frac{8}{3}y = -\frac{2}{3}}$; $\boxed{y^2 - \frac{8}{3}y + \left(-\frac{\frac{\overset{4}{\cancel{8}}}{3}}{\underset{3}{\cancel{6}}}\right)^2 = -\frac{2}{3} + \left(-\frac{\frac{\overset{4}{\cancel{8}}}{3}}{\underset{3}{\cancel{6}}}\right)^2}$

; $\boxed{y^2-\frac{8}{3}y+\left(\frac{4}{3}\right)^2=-\frac{2}{3}+\left(\frac{4}{3}\right)^2}$; $\boxed{y^2-\frac{8}{3}y+\frac{16}{9}=-\frac{2}{3}+\frac{16}{9}}$; $\boxed{\left(y-\frac{4}{3}\right)^2=-\frac{2}{3}+\frac{16}{9}}$

; $\boxed{\left(y-\frac{4}{3}\right)^2=\frac{(-2\cdot 9)+(16\cdot 3)}{3\cdot 9}}$; $\boxed{\left(y-\frac{4}{3}\right)^2=\frac{-18+48}{27}}$; $\boxed{\left(y-\frac{4}{3}\right)^2=\frac{30}{27}}$; $\boxed{\sqrt{\left(y-\frac{4}{3}\right)^2}=\pm\sqrt{\frac{30}{27}}}$

; $\boxed{y-\frac{4}{3}=\pm\sqrt{\frac{30}{27}}}$; $\boxed{y-\frac{4}{3}=\pm\sqrt{1.11}}$; $\boxed{y-1.33=\pm 1.05}$ therefore:

I. $\boxed{y-1.33=+1.05}$; $\boxed{y=1.05+1.33}$; $\boxed{\mathbf{y=2.38}}$ II. $\boxed{y-1.33=-1.05}$; $\boxed{y=-1.05+1.33}$; $\boxed{\mathbf{y=0.28}}$

and the solution set is $\{\mathbf{0.28}, \mathbf{2.38}\}$.

Check: I. *Let* $y=0.28$ *in* $3y^2-8y+2=0$; $3\cdot(0.28)^2-8\cdot 0.28+2\overset{?}{=}0$; $0.24-2.24+2\overset{?}{=}0$

; $2.24-2.24\overset{?}{=}0$; $0=0$

II. *Let* $y=2.38$ *in* $3y^2-8y+2=0$; $3\cdot(2.38)^2-8\cdot 2.38+2\overset{?}{=}0$; $17-19+2\overset{?}{=}0$; $0=0$

Therefore, the equation $3y^2-8y+2=0$ can be factored to $(\boldsymbol{y}-\mathbf{0.28})(\boldsymbol{y}-\mathbf{2.38})=\mathbf{0}$.

Example 1.4-18

Solve the quadratic equation $3t^2+12t-4=0$ using Completing-the-Square method.

Solution:

$\boxed{3t^2+12t-4=0}$; $\boxed{3t^2+12t=4}$; $\boxed{\frac{\cancel{3}}{\cancel{3}}t^2+\frac{\overset{4}{\cancel{12}}}{\cancel{3}}x=\frac{4}{3}}$; $\boxed{t^2+4t=\frac{4}{3}}$; $\boxed{t^2+4t+\left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)^2=\frac{4}{3}+\left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)^2}$

; $\boxed{t^2+4t+\left(\frac{2}{1}\right)^2=\frac{4}{3}+\left(\frac{2}{1}\right)^2}$; $\boxed{t^2+4t+4=\frac{4}{3}+4}$; $\boxed{(t+2)^2=\frac{4}{3}+\frac{4}{1}}$; $\boxed{(t+2)^2=\frac{(4\cdot 1)+(4\cdot 3)}{3\cdot 1}}$

; $\boxed{(t+2)^2=\frac{4+12}{3}}$; $\boxed{(t+2)^2=\frac{16}{3}}$; $\boxed{\sqrt{(t+2)^2}=\pm\sqrt{\frac{16}{3}}}$; $\boxed{t+2=\pm\sqrt{5.33}}$; $\boxed{t+2=\pm 2.31}$ therefore:

I. $\boxed{t+2=+2.31}$; $\boxed{t=2.31-2}$; $\boxed{\mathbf{t=0.31}}$ II. $\boxed{t+2=-2.31}$; $\boxed{t=-2.31-2}$; $\boxed{\mathbf{t=-4.31}}$

and the solution set is $\{\mathbf{0.31}, -\mathbf{4.31}\}$.

Check: I. *Let* $t=0.31$ *in* $3t^2+12t-4=0$; $3\cdot(0.31)^2+(12\cdot 0.31)-4\overset{?}{=}0$; $3\cdot 0.096-3.72-4\overset{?}{=}0$

; $0.288+3.72-4\overset{?}{=}0$; $4-4\overset{?}{=}0$; $0=0$

II. *Let* $t=-4.31$ *in* $3t^2+12t-4=0$; $3\cdot(-4.31)^2+(12\cdot -4.31)-4\overset{?}{=}0$; $3\cdot 18.57-51.72-4\overset{?}{=}0$

; $55.72-51.72-4\overset{?}{=}0$; $55.72-55.72\overset{?}{=}0$; $0=0$

Therefore, the equation $3t^2+12t-4=0$ can be factored to $(\boldsymbol{t}-\mathbf{0.31})(\boldsymbol{t}+\mathbf{4.31})=\mathbf{0}$.

Example 1.4-19

Solve the quadratic equation $2a^2+16a-6=0$ using Completing-the-Square method.

Solution:

$\boxed{2a^2+16a-6=0}$; $\boxed{2a^2+16a=6}$; $\boxed{\frac{\cancel{2}}{\cancel{2}}a^2+\frac{\overset{8}{\cancel{16}}}{\cancel{2}}a=\frac{\overset{3}{\cancel{6}}}{\cancel{2}}}$; $\boxed{a^2+8a=3}$; $\boxed{a^2+8a+\left(\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2=3+\left(\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2}$

; $\boxed{a^2+8a+\left(\frac{4}{1}\right)^2=3+\left(\frac{4}{1}\right)^2}$; $\boxed{a^2+8a+16=3+16}$; $\boxed{(a+4)^2=19}$; $\boxed{\sqrt{(a+4)^2}=\pm\sqrt{19}}$; $\boxed{a+4=\pm 4.36}$

Therefore:

I. $\boxed{a+4=+4.36}$; $\boxed{a=4.36-4}$; $\boxed{\mathbf{a=0.36}}$ II. $\boxed{a+4=-4.36}$; $\boxed{a=-4.36-4}$; $\boxed{\mathbf{a=-8.36}}$

and the solution set is $\{\mathbf{-8.36,\ 0.36}\}$.

Check: I. *Let* $a=-8.36$ *in* $2a^2+16a-6=0$; $2\cdot(-8.36)^2+(16\cdot -8.36)-6\overset{?}{=}0$; $2\cdot 69.9-133.8-6\overset{?}{=}0$

; $139.8-133.8-6\overset{?}{=}0$; $139.8-139.8\overset{?}{=}0$; $0=0$

II. *Let* $a=0.36$ *in* $2a^2+16a-6=0$; $2\cdot(0.36)^2+(16\cdot 0.36)-6\overset{?}{=}0$; $2\cdot 0.129+5.7-6\overset{?}{=}0$

; $0.3+5.7-6\overset{?}{=}0$; $6-6\overset{?}{=}0$; $0=0$

Therefore, the equation $2a^2+16a-6=0$ can be factored to $(\boldsymbol{a}+\mathbf{8.36})(\boldsymbol{a}-\mathbf{0.36})=\mathbf{0}$.

Example 1.4-20

Solve the quadratic equation $4n^2+5n-2=0$ using Completing-the-Square method.

Solution:

$\boxed{4n^2+5n-2=0}$; $\boxed{4n^2+5n=2}$; $\boxed{\frac{\cancel{4}}{\cancel{4}}n^2+\frac{5}{4}n=\frac{\cancel{2}}{\underset{2}{\cancel{4}}}}$; $\boxed{n^2+\frac{5}{4}n=\frac{1}{2}}$; $\boxed{n^2+\frac{5}{4}n+\left(\frac{5}{8}\right)^2=\frac{1}{2}+\left(\frac{5}{8}\right)^2}$

; $\boxed{n^2+\frac{5}{4}n+\frac{25}{64}=\frac{1}{2}+\frac{25}{64}}$; $\boxed{\left(n+\frac{5}{8}\right)^2=\frac{(1\cdot 64)+(25\cdot 2)}{2\cdot 64}}$; $\boxed{\left(n+\frac{5}{8}\right)^2=\frac{64+50}{128}}$; $\boxed{\left(n+\frac{5}{8}\right)^2=\frac{114}{128}}$

; $\boxed{\sqrt{\left(n+\frac{5}{8}\right)^2}=\pm\sqrt{\frac{114}{128}}}$; $\boxed{n+\frac{5}{8}=\pm\sqrt{0.89}}$; $\boxed{n+0.63=\pm 0.94}$ therefore:

I. $\boxed{n+0.63=+0.94}$; $\boxed{n=0.94-0.63}$; $\boxed{\mathbf{n=0.31}}$ II. $\boxed{n+0.63=-0.94}$; $\boxed{n=-0.94-0.63}$; $\boxed{\mathbf{n=-1.6}}$

and the solution set is $\{\mathbf{-1.6,\ 0.31}\}$.

Check: I. *Let* $n=-1.6$ *in* $4n^2+5n-2=0$; $4\cdot(-1.6)^2+(5\cdot -1.6)-2\overset{?}{=}0$; $4\cdot 2.5-8-2\overset{?}{=}0$; $0=0$

II. *Let* $n=0.31$ *in* $4n^2+5n-2=0$; $4\cdot 0.31^2+(5\cdot 0.31)-2\overset{?}{=}0$; $4\cdot 0.1+1.6-2\overset{?}{=}0$

; $0.4+1.6-2\overset{?}{=}0$; $2-2\overset{?}{=}0$; $0=0$

Therefore, the equation $4n^2+5n-2=0$ can be factored to $(\boldsymbol{n}-\mathbf{0.31})(\boldsymbol{n}+\mathbf{1.6})=\mathbf{0}$.

Practice Problems - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a \rangle 1$, by Completing the Square

Section 1.4 Case II Practice Problems - Solve the following quadratic equations using Completing-the-Square method. (Note that these problems are identical to the exercises given in Section 1.2 Case II.)

1. $4u^2 + 6u + 1 = 0$
2. $4w^2 + 10w = -3$
3. $6x^2 + 4x - 2 = 0$
4. $15y^2 + 3 = -14y$
5. $2x^2 - 5x + 3 = 0$
6. $2x^2 + xy - y^2 = 0$ x is variable
7. $6x^2 + 7x - 3 = 0$
8. $5x^2 = -3x$
9. $3x^2 + 4x + 5 = 0$
10. $-3y^2 + 13y + 10 = 0$

1.5 Solving Other Types of Quadratic Equations

In this section two classes of quadratic equations are addressed: one containing radicals (Case I) and the second containing fractions (Case II).

Case I - Solving Quadratic Equations Containing Radicals

In general, radical equations are solved by squaring both sides of the equation. This squaring process sometimes produces solutions that when substituted into the original equation do not produce equality in both sides of the equation. These solutions are called apparent solutions. For example, the equation $y = 7$ has only one solution, i.e., 7. Let's square both sides of the equation $y = 7$ to obtain $y^2 = 49$ and solve for y by taking square root of both sides, i.e., $\sqrt{y^2} = \pm\sqrt{49}$. Solving for y we obtain $y = \pm 7$. However, note that by substituting the two solutions into the original equation $y = 7$, it is clear that only $y = +7$ is the real solution and $y = -7$ is the apparent solution. Therefore, in order to identify the real solutions **we must check all solutions in the original equation**. The following show the steps as to how equations containing radical expressions are solved:

Step 1 Square both sides of the equation.

Step 2 Write the quadratic equation in standard form.

Step 3 Solve the quadratic equation by choosing a solution method.

Step 4 Check the answers by substituting the solutions into the original equation. Disregard any apparent solution.

Examples with Steps

The following examples show the steps as to how equations containing radicals are solved:

Example 1.5-1

Solve the radical equation $\sqrt{x^2+5} = 3$.

Solution:

Step 1 $\boxed{\sqrt{x^2+5} = 3}$; $\boxed{\left(\sqrt{x^2+5}\right)^2 = 3^2}$; $\boxed{x^2+5 = 9}$

Step 2 $\boxed{x^2+5 = 9}$; $\boxed{x^2+5-9 = 9-9}$; $\boxed{x^2-4 = 0}$

Step 3 $\boxed{x^2-4 = 0}$; $\boxed{x^2 = 4}$; $\boxed{\sqrt{x^2} = \pm\sqrt{4}}$; $\boxed{x = \pm 2}$ (Solve using the Square Root Property method)

Therefore, the two apparent solutions are $x = -2$ and $x = 2$.

Step 4 Check: Substitute $x = 2$ and $x = -2$ in $\sqrt{x^2+5} = 3$.

I. *Let* $x = 2$ *in* $\sqrt{x^2+5} = 3$; $\sqrt{2^2+5} \stackrel{?}{=} 3$; $\sqrt{4+5} \stackrel{?}{=} 3$; $\sqrt{9} \stackrel{?}{=} 3$; $3 = 3$

II. *Let* $x = -2$ *in* $\sqrt{x^2+5} = 3$; $\sqrt{(-2)^2+5} \stackrel{?}{=} 3$; $\sqrt{4+5} \stackrel{?}{=} 3$; $\sqrt{9} \stackrel{?}{=} 3$; $3 = 3$

Thus, $\boldsymbol{x = -2}$ and $\boldsymbol{x = 2}$ are the real solutions to $\sqrt{x^2+5} = 3$. Furthermore, the equation $\sqrt{x^2+5} = 3$ can be factored to $\boldsymbol{(x-2)(x+2) = 0}$.

Example 1.5-2

Solve the radical equation $\sqrt{-12x-4} = 3x+1$.

Solution:

Step 1 $\boxed{\sqrt{-12x-4} = 3x+1}$; $\boxed{\left(\sqrt{-12x-4}\right)^2 = (3x+1)^2}$; $\boxed{-12x-4 = (3x+1)^2}$

Step 2 $\boxed{-12x-4 = (3x+1)^2}$; $\boxed{-12x-4 = 9x^2+1+6x}$; $\boxed{9x^2+1+6x+12x+4 = 0}$

; $\boxed{9x^2+(6x+12x)+(4+1) = 0}$; $\boxed{9x^2+18x+5 = 0}$

Step 3 $\boxed{9x^2+18x+5 = 0}$; $\boxed{\left(x+\frac{1}{3}\right)\left(x+\frac{5}{3}\right)}$ (Solve using the Quadratic Formula)

Therefore, the two apparent solutions are $x = -\frac{1}{3}$ and $x = -\frac{5}{3}$.

Step 4 Check: Substitute $x = -\frac{1}{3}$ and $x = -\frac{5}{3}$ in $\sqrt{-12x-4} = 3x+1$.

I. *Let* $x = -\frac{1}{3}$ *in* $\sqrt{-12x-4} = 3x+1$; $\sqrt{-\overset{4}{\cancel{12}}\cdot\left(-\frac{1}{\cancel{3}}\right)-4} \stackrel{?}{=} \cancel{3}\cdot\left(-\frac{1}{\cancel{3}}\right)+1$; $\sqrt{4-4} \stackrel{?}{=} -1+1$

; $\sqrt{0} \stackrel{?}{=} 0$; $0 = 0$

II. *Let* $x = -\frac{5}{3}$ *in* $\sqrt{-12x-4} = 3x+1$; $\sqrt{-\overset{4}{\cancel{12}}\cdot\left(-\frac{5}{\cancel{3}}\right)-4} \stackrel{?}{=} \cancel{3}\cdot\left(-\frac{5}{\cancel{3}}\right)+1$

; $\sqrt{-4\cdot(-5)-4} \stackrel{?}{=} -5+1$; $\sqrt{20-4} \stackrel{?}{=} -4$; $\sqrt{16} \stackrel{?}{=} -4$; $\sqrt{4^2} \stackrel{?}{=} -4$; $4 \neq -4$

Thus, the equation $\sqrt{-12x-4} = 3x+1$ has one real solution, i.e., $\boldsymbol{x = -\frac{1}{3}}$.

Example 1.5-3

Solve the radical equation $x+1 = \sqrt{x+1}$.

Solution:

Step 1 $\boxed{x+1 = \sqrt{x+1}}$; $\boxed{(x+1)^2 = \left(\sqrt{x+1}\right)^2}$; $\boxed{x^2+1+2x = x+1}$

Step 2 $\boxed{x^2+1+2x = x+1}$; $\boxed{x^2+(2x-x)+(1-1) = 0}$; $\boxed{x^2+x+0 = 0}$; $\boxed{x^2+x = 0}$

Step 3 $\boxed{x^2+x = 0}$; $\boxed{x(x+1) = 0}$

Therefore, the two apparent solutions are $x = 0$ and $x = -1$.

Step 4 Check: Substitute $x = 0$ and $x = -1$ in $x+1 = \sqrt{x+1}$.

I. *Let* $x = 0$ *in* $x+1=\sqrt{x+1}$; $0+1\overset{?}{=}\sqrt{0+1}$; $1\overset{?}{=}\sqrt{1}$; $1=1$

II. *Let* $x=-1$ *in* $x+1=\sqrt{x+1}$; $-1+1\overset{?}{=}\sqrt{-1+1}$; $0\overset{?}{=}\sqrt{0}$; $0=0$

Thus, $\boldsymbol{x=0}$ and $\boldsymbol{x=-1}$ are the real solutions to $x+1=\sqrt{x+1}$. Furthermore, the equation $x+1=\sqrt{x+1}$ can be factored to $\boldsymbol{x(x+1)=0}$.

Example 1.5-4

Solve the radical equation $2t=\sqrt{11t-6}$.

Solution:

Step 1 $\boxed{2t=\sqrt{11t-6}}$; $\boxed{(2t)^2=\left(\sqrt{11t-6}\right)^2}$; $\boxed{4t^2=11t-6}$

Step 2 $\boxed{4t^2=11t-6}$; $\boxed{4t^2-11t+6=0}$

Step 3 $\boxed{4t^2-11t+6=0}$; $\boxed{\left(t-\frac{3}{4}\right)(t-2)}$ (Solve Using the Quadratic Formula)

Therefore, the two apparent solutions are $t=\frac{3}{4}$ and $t=2$.

Step 4 Check: Substitute $t=\frac{3}{4}$ and $t=2$ in $2t=\sqrt{11t-6}$.

I. *Let* $t=\frac{3}{4}$ *in* $2t=\sqrt{11t-6}$; $\cancel{2}\cdot\frac{3}{\underset{2}{\cancel{4}}}\overset{?}{=}\sqrt{11\cdot\frac{3}{4}-6}$; $\frac{3}{2}\overset{?}{=}\sqrt{\frac{33}{4}-\frac{6}{1}}$; $\frac{3}{2}\overset{?}{=}\sqrt{\frac{(33\cdot1)-(6\cdot4)}{4\cdot1}}$

; $\frac{3}{2}\overset{?}{=}\sqrt{\frac{33-24}{4}}$; $\frac{3}{2}\overset{?}{=}\sqrt{\frac{9}{4}}$; $\frac{3}{2}\overset{?}{=}\sqrt{\frac{3^2}{2^2}}$; $\frac{3}{2}=\frac{3}{2}$

II. *Let* $t=2$ *in* $2t=\sqrt{11t-6}$; $2\cdot2\overset{?}{=}\sqrt{11\cdot2-6}$; $4\overset{?}{=}\sqrt{22-6}$; $4\overset{?}{=}\sqrt{16}$; $4=4$

Thus, $\boldsymbol{t=\frac{3}{4}}$ and $\boldsymbol{t=2}$ are the real solutions to $2t=\sqrt{11t-6}$. Furthermore, the equation $2t=\sqrt{11t-6}$ can be factored to $\left(t-\frac{3}{4}\right)(t-2)=0$ which is the same as $\boldsymbol{(4t-3)(t-2)=0}$

Example 1.5-5

Solve the radical equation $\sqrt{2}w=\sqrt{3w-1}$.

Solution:

Step 1 $\boxed{\sqrt{2}w=\sqrt{3w-1}}$; $\boxed{\left(\sqrt{2}w\right)^2=\left(\sqrt{3w-1}\right)^2}$; $\boxed{2w^2=3w-1}$

Step 2 $\boxed{2w^2=3w-1}$; $\boxed{2w^2-3w+1=0}$

Step 3 $\boxed{2w^2 - 3w + 1 = 0}$; $\boxed{\left(w - \frac{1}{2}\right)(w-1) = 0}$ (Use Completing-the-Square method)

Therefore, the two apparent solutions are $w = \frac{1}{2}$ and $w = 1$.

Step 4 Check: Substitute $w = \frac{1}{2}$ and $w = 1$ in $\sqrt{2}w = \sqrt{3w-1}$.

I. *Let* $w = \frac{1}{2}$ *in* $\sqrt{2}w = \sqrt{3w-1}$; $\sqrt{2} \cdot \frac{1}{2} \stackrel{?}{=} \sqrt{3 \cdot \frac{1}{2} - 1}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \sqrt{\frac{3}{2} - \frac{1}{1}}$

; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \sqrt{\frac{(3 \cdot 1) - (1 \cdot 2)}{2 \cdot 1}}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \sqrt{\frac{3-2}{2}}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \sqrt{\frac{1}{2}}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \frac{\sqrt{1}}{\sqrt{2}}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \frac{\sqrt{1} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \frac{\sqrt{1 \cdot 2}}{\sqrt{2 \cdot 2}}$; $\frac{\sqrt{2}}{2} \stackrel{?}{=} \frac{\sqrt{2}}{\sqrt{2^2}}$; $\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

II. *Let* $w = 1$ *in* $\sqrt{2}w = \sqrt{3w-1}$; $\sqrt{2} \cdot 1 \stackrel{?}{=} \sqrt{3 \cdot 1 - 1}$; $\sqrt{2} \stackrel{?}{=} \sqrt{3-1}$; $\sqrt{2} = \sqrt{2}$

Thus, $\boldsymbol{w = \frac{1}{2}}$ and $\boldsymbol{w = 1}$ are the real solutions to $\sqrt{2}w = \sqrt{3w-1}$. Furthermore, the equation $\sqrt{2}w = \sqrt{3w-1}$ can be factored to $(w-1)\left(w - \frac{1}{2}\right) = 0$ which is the same as $\boldsymbol{(w-1)(2w-1) = 0}$.

Additional Examples - Solving Quadratic Equations Containing Radicals

The following examples further illustrate how to solve quadratic equations that contain radical expressions:

Example 1.5-6

Solve the radical equation $x + 4 = \sqrt{x+4}$.

Solution:

First - Square both sides of the equation. $x + 4 = \sqrt{x+4}$; $(x+4)^2 = \left(\sqrt{x+4}\right)^2$; $(x+4)^2 = x + 4$

Second - Complete the square on the left hand side of the equation and simplify.

$(x+4)^2 = x + 4$; $x^2 + 16 + 8x = x + 4$; $x^2 + 16 - 4 + 8x - x = 0$; $x^2 + 12 + 7x = 0$

Third - Write the quadratic equation in standard form. $x^2 + 7x + 12 = 0$

Fourth - Solve the quadratic equation by choosing a solution method.

$x^2 + 7x + 12 = 0$; $(x+4)(x+3) = 0$. Therefore, the two apparent solutions are:

$x + 4 = 0$; $x = -4$ and $x + 3 = 0$; $x = -3$

Fifth - Check the answers by substituting the x values into the original equation.

I. *Let* $x = -4$ *in* $x + 4 = \sqrt{x+4}$; $-4 + 4 \stackrel{?}{=} \sqrt{-4+4}$; $0 = 0$

II. *Let* $x = -3$ *in* $x + 4 = \sqrt{x+4}$; $-3 + 4 \stackrel{?}{=} \sqrt{-3+4}$; $1 \stackrel{?}{=} \sqrt{1}$; $1 = 1$

Therefore, $\boldsymbol{x=-4}$ and $\boldsymbol{x=-3}$ are the real solutions to $x+4=\sqrt{x+4}$. Furthermore, the equation $x+4=\sqrt{x+4}$ can be factored to $\boldsymbol{(x+4)(x+3)=0}$.

Example 1.5-7

Solve the radical equation $\sqrt{3x+4}=x$.

Solution:

First - Square both sides of the equation. $\sqrt{3x+4}=x$; $\left(\sqrt{3x+4}\right)^2=x^2$; $3x+4=x^2$

Second - Write the quadratic equation in standard form. $x^2-3x-4=0$

Third - Solve the quadratic equation by choosing a solution method.

$x^2-3x-4=0$; $(x-4)(x+1)=0$. Therefore, the two apparent solutions are:

$x-4=0$; $x=4$ and $x+1=0$; $x=-1$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=4$ *in* $\sqrt{3x+4}=x$; $\sqrt{3\cdot 4+4}\stackrel{?}{=}4$; $\sqrt{12+4}\stackrel{?}{=}4$; $\sqrt{16}\stackrel{?}{=}4$; $\sqrt{4^2}\stackrel{?}{=}4$; $4=4$

II. *Let* $x=-1$ *in* $\sqrt{3x+4}=x$; $\sqrt{(3\cdot -1)+4}\stackrel{?}{=}-1$; $\sqrt{-3+4}\stackrel{?}{=}-1$; $\sqrt{1}\stackrel{?}{=}-1$; $1\neq -1$

Therefore, the equation $\sqrt{3x-4}=x$ has one real solution, i.e., $\boldsymbol{x=4}$.

Example 1.5-8

Solve the radical equation $\sqrt{u^2+5}=u+2$.

Solution:

First - Square both sides of the equation. $\sqrt{u^2+5}=u+2$; $\left(\sqrt{u^2+5}\right)^2=(u+2)^2$; $u^2+5=(u+2)^2$

Second - Complete the square on the right hand side of the equation and simplify.

$u^2+5=u^2+4+4u$; $u^2-u^2+4-5+4u=0$; $-1+4u=0$

Third - Solve for u, i.e., $-1+4u=0$; $4u=1$; $u=\frac{1}{4}$; $u=0.25$

Fourth - Check the answers by substituting the u solution into the original equation, i.e.,

Let $u=0.25$ *in* $\sqrt{u^2+5}=u+2$; $\sqrt{0.25^2+5}\stackrel{?}{=}0.25+2$; $\sqrt{0.0625+5}\stackrel{?}{=}2.25$; $\sqrt{5.0625}\stackrel{?}{=}2.25$; $2.25=2.25$. Therefore, the equation $\sqrt{u^2+5}=u+2$ has one solution, i.e., $\boldsymbol{u=0.25}$.

Example 1.5-9

Solve the radical equation $\sqrt{2x+15}=x$.

Solution:

First - Square both sides of the equation. $\sqrt{2x+15}=x$; $\left(\sqrt{2x+15}\right)^2=x^2$; $2x+15=x^2$

Second - Write the quadratic equation in standard form. $x^2-2x-15=0$

Third - Solve the quadratic equation by choosing a solution method.

$x^2-2x-15=0$; $(x-5)(x+3)=0$. Therefore, the two apparent solutions are:

$x-5=0$; $x=5$ and $x+3=0$; $x=-3$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=5$ *in* $\sqrt{2x+15}=x$; $\sqrt{2\cdot 5+15}\stackrel{?}{=}5$; $\sqrt{10+15}\stackrel{?}{=}5$; $\sqrt{25}\stackrel{?}{=}5$; $5=5$

II. *Let* $x=-3$ *in* $\sqrt{2x+15}=x$; $\sqrt{2\cdot -3+15}\overset{?}{=}-3$; $\sqrt{-6+15}\overset{?}{=}-3$; $\sqrt{9}\overset{?}{=}-3$; $3\neq-3$

Therefore, the equation $\sqrt{2x+15}=x$ has one real solution, i.e., $\mathbf{x=5}$.

Example 1.5-10

Solve the radical equation $\sqrt{x+30}=x$.

Solution:

First - Square both sides of the equation. $\sqrt{x+30}=x$; $\left(\sqrt{x+30}\right)^2=x^2$; $x+30=x^2$

Second - Write the quadratic equation in standard form. $x^2-x-30=0$

Third - Solve the quadratic equation by choosing a solution method.

$x^2-x-30=0$; $(x-6)(x+5)=0$. Therefore, the two apparent solutions are:

$x-6=0$; $x=6$ and $x+5=0$; $x=-5$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=6$ *in* $\sqrt{x+30}=x$; $\sqrt{6+30}\overset{?}{=}6$; $\sqrt{36}\overset{?}{=}6$; $6=6$

II. *Let* $x=-5$ *in* $\sqrt{x+30}=x$; $\sqrt{-5+30}\overset{?}{=}-5$; $\sqrt{25}\overset{?}{=}-5$; $5\neq-5$

Therefore, the equation $\sqrt{x+30}=x$ has one real solution, i.e., $\mathbf{x=6}$.

Practice Problems - Solving Quadratic Equations Containing Radicals

Section 1.5 Case I Practice Problems - Solve the following equations. Check the answers by substituting the solutions into the original equation.

1. $\sqrt{-9y+28}-y+2=0$
2. $2x=\sqrt{9x+3}$
3. $t^2=-\sqrt{5}t$
4. $y^2-\sqrt{8}y=7$
5. $\sqrt{5}x=2x^2$
6. $\sqrt{x^2-12}=2$
7. $\sqrt{-8x-4}=2x+1$
8. $x=\sqrt{-x+2}$
9. $x=\sqrt{-2x+3}$
10. $\sqrt{x^2+3}=x+1$

Case II - Solving Quadratic Equations Containing Fractions

In this section solutions to quadratic equations with fractional coefficients are discussed. Note that in dealing with fractional equations not all solutions may satisfy the original equation. This is because fractions may encounter division by zero which is undefined. Therefore, it is essential that all solutions to a quadratic equation be checked by substitution into the original equation in order to ensure division by zero does not occur. Equations containing algebraic fractions are solved using the following steps:

Step 1 Write both sides of the equation in fraction form.

Step 2 Cross multiply the terms in both sides of the equation.

Step 3 Write the quadratic equation in standard form.

Step 4 Solve the quadratic equation by choosing a solution method.

Step 5 Check the answers by substituting the apparent solutions into the original equation. Disregard any apparent solution if equality on both sides of the equation is not obtained.

Examples with Steps

The following examples show the steps as to how equations containing fractions are solved:

Example 1.5-11

Solve the fractional equation $x-1=\frac{20}{x}$.

Solution:

First - Write the left hand side of the equation in fraction form. $x-1=\frac{20}{x}$; $\frac{x-1}{1}=\frac{20}{x}$

Second - Cross multiply the terms in both sides of the equation. $x\cdot(x-1)=1\cdot 20$; $x^2-x=20$

Third - Write the quadratic equation in standard form, i.e., $x^2-x-20=0$

Fourth - Solve the quadratic equation by choosing a method. $x^2-x-20=0$; $(x-5)(x+4)=0$.

Therefore, the two apparent solutions are: $x-5=0$; $x=5$ and $x+4=0$; $x=-4$

Fifth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=5$ *in* $x-1=\frac{20}{x}$; $5-1\stackrel{?}{=}\frac{20}{5}$; $4=4$

II. *Let* $x=-4$ *in* $x-1=\frac{20}{x}$; $-4-1\stackrel{?}{=}\frac{20}{-4}$; $-5=-5$

Thus, $\boldsymbol{x=5}$ and $\boldsymbol{x=-4}$ are the real solutions to $x-1=\frac{20}{x}$. In addition, the equation $x-1=\frac{20}{x}$ can be factored to $\boldsymbol{(x-5)(x+4)=0}$.

Example 1.5-12

Solve the fractional equation $1+\frac{1}{x+1}=x+3$.

Solution:

First - use fraction techniques to rewrite the left hand side of the equation in a single fraction form.

$$1+\frac{1}{x+1}=x+3\ ;\ \frac{1}{1}+\frac{1}{x+1}=\frac{x+3}{1}\ ;\ \frac{[1\cdot(x+1)]+(1\cdot 1)}{1\cdot(x+1)}=\frac{x+3}{1}\ ;\ \frac{x+1+1}{x+1}=\frac{x+3}{1}\ ;\ \frac{x+2}{x+1}=\frac{x+3}{1}$$

Second - Cross multiply the terms in both sides of the equation.

$$\frac{x+2}{x+1}=\frac{x+3}{1}\ ;\ (x+2)\cdot 1=(x+3)\cdot(x+1)\ ;\ x+2=x^2+x+3x+3\ ;\ x+2=x^2+4x+3$$

Third - write the quadratic equation in standard form, i.e., $x^2+4x-x+3-2=0\ ;\ x^2+3x+1=0$

Fourth - Solve the quadratic equation using the Quadratic Formula method.

$$x^2+3x+1=0\ ;\ \left(x+\frac{3-\sqrt{5}}{2}\right)\left(x+\frac{3+\sqrt{5}}{2}\right)=0\ ;\ \left(x+\frac{3-2.24}{2}\right)\left(x+\frac{3+2.24}{2}\right)=0$$

$$;\ (x+0.38)(x+2.62)=0\,.$$

Therefore, the two apparent solutions are: $x+0.38=0\ ;\ x=-0.38$ and $x+2.62=0\ ;\ x=-2.62$

Fifth - Check the answers by substituting the x values into the original equation.

I. $Let\ x=-0.38\ \ in\ \ 1+\frac{1}{x+1}=x+3\ ;\ 1+\frac{1}{-0.38+1}\stackrel{?}{=}-0.38+3\ ;\ 1+\frac{1}{0.62}\stackrel{?}{=}2.62\ ;\ 1+1.62\stackrel{?}{=}2.62$

$;\ 2.62=2.62$

II. $Let\ x=-2.62\ \ in\ \ 1+\frac{1}{x+1}=x+3\ ;\ 1+\frac{1}{-2.62+1}\stackrel{?}{=}-2.62+3\ ;\ 1-\frac{1}{1.62}\stackrel{?}{=}0.38\ ;\ 1-0.62\stackrel{?}{=}0.38$

$;\ 0.38=0.38$

Thus, $\boldsymbol{x=-0.38}$ and $\boldsymbol{x=-2.62}$ are the real solutions to $1+\frac{1}{x+1}=x+3$. In addition, the equation $1+\frac{1}{x+1}=x+3$ can be factored to $\boldsymbol{(x+0.38)(x+2.62)=0}$.

Example 1.5-13

Solve the fractional equation $\frac{1}{2y}=6y-\frac{1}{y}$.

Solution:

First - Write the left hand side of the equation in fraction form and simplify the right hand side of the equation.

$$\frac{1}{2y}=6y-\frac{1}{y}\ ;\ \frac{1}{2y}=\frac{6y}{1}-\frac{1}{y}\ ;\ \frac{1}{2y}=\frac{(6y\cdot y)-(1\cdot 1)}{1\cdot y}\ ;\ \frac{1}{2y}=\frac{6y^2-1}{y}$$

Second - Cross multiply the terms in both sides of the equation.

$$\frac{1}{2y}=\frac{6y^2-1}{y}\ ;\ 1\cdot y=2y\cdot\left(6y^2-1\right)\ ;\ y=12y^3-2y\ ;\ 12y^3-2y-y=0\ ;\ 12y^3-3y=0$$

$;\ 3y\left(4y^2-1\right)=0$. Thus, $y=0$ is an apparent solution.

Third - Solve the quadratic equation $4y^2-1=0$ by choosing a method.

$4y^2-1=0\ ;\ 4y^2=1\ ;\ \sqrt{4y^2}=\pm\sqrt{1}\ ;\ 2y=\pm 1\ ;\ y=\pm\frac{1}{2}$. Therefore, the other two apparent solutions are: $y=+\frac{1}{2}\ ;\ y=+0.5$ and $y=-\frac{1}{2}\ ;\ y=-0.5$.

Fourth - Check the answers by substituting the y values into the original equation.

I. *Let* $y = 0$ *in* $\frac{1}{2y} = 6y - \frac{1}{y}$. Since division by zero is encountered therefore, $y = 0$ is not a real solution.

II. *Let* $y = 0.5$ *in* $\frac{1}{2y} = 6y - \frac{1}{y}$; $\frac{1}{2 \times 0.5} \stackrel{?}{=} 6 \times 0.5 - \frac{1}{0.5}$; $\frac{1}{1} \stackrel{?}{=} 3 - 2$; $1 = 1$

III. *Let* $y = -0.5$ *in* $\frac{1}{2y} = 6y - \frac{1}{y}$; $\frac{1}{(2 \times -0.5)} \stackrel{?}{=} (6 \times -0.5) - \frac{1}{(-0.5)}$; $\frac{1}{-1} \stackrel{?}{=} -3 + 2$; $-1 = -1$

Thus, $\mathbf{y = 0.5}$ and $\mathbf{y = -0.5}$ are the real solutions to $\frac{1}{2y} = 6y - \frac{1}{y}$. In addition, the equation $\frac{1}{2y} = 6y - \frac{1}{y}$ can be factored to $\mathbf{3y(y + 0.5)(y - 0.5) = 0}$.

Example 1.5-14

Solve the fractional equation $\frac{2}{y} + \frac{3}{y^2} - 1 = 0$.

Solution:

First - Use fraction techniques to simplify the left hand side of the equation.

$\frac{2}{y} + \frac{3}{y^2} - 1 = 0$; $\frac{\left(2 \cdot y^2\right) + (3 \cdot y)}{y \cdot y^2} - 1 = 0$; $\frac{2y^2 + 3y}{y^3} - \frac{1}{1} = 0$; $\frac{1 \cdot \left(2y^2 + 3y\right) - 1 \cdot y^3}{1 \cdot y^3} = 0$; $\frac{2y^2 + 3y - y^3}{y^3} = 0$

Second - Cross multiply the terms in both sides of the equation.

$\frac{2y^2 + 3y - y^3}{y^3} = \frac{0}{1}$; $1 \cdot \left(2y^2 + 3y - y^3\right) = 0 \cdot y^3$; $2y^2 + 3y - y^3 = 0$; $-y\left(-2y - 3 + y^2\right) = 0$. Thus $y = 0$ is an apparent solution.

Third - Write the quadratic equation $-2y - 3 + y^2 = 0$ in standard form, i.e., $y^2 - 2y - 3 = 0$

Fourth - Solve the quadratic equation by choosing a method. $y^2 - 2y - 3 = 0$; $(y - 3)(y + 1) = 0$.

Therefore, the other two apparent solutions are: $y - 3 = 0$; $y = 3$ and $y + 1 = 0$; $y = -1$

Fifth - Check the answers by substituting the y values into the original equation.

I. *Let* $y = 0$ *in* $\frac{2}{y} + \frac{3}{y^2} - 1 = 0$. Since division by zero is encountered therefore, $y = 0$ is not a real solution.

II. *Let* $y = 3$ *in* $\frac{2}{y} + \frac{3}{y^2} - 1 = 0$; $\frac{2}{3} + \frac{3}{3^2} - 1 \stackrel{?}{=} 0$; $\frac{2}{3} + \frac{\cancel{3}}{\underset{3}{\cancel{9}}} - 1 \stackrel{?}{=} 0$; $\frac{2}{3} + \frac{1}{3} - 1 \stackrel{?}{=} 0$; $\frac{2 + 1}{3} - 1 \stackrel{?}{=} 0$; $\frac{\cancel{3}}{\cancel{3}} - 1 \stackrel{?}{=} 0$; $\frac{1}{1} - 1 \stackrel{?}{=} 0$; $1 - 1 = 0$; $0 = 0$

III. *Let* $y = -1$ *in* $\frac{2}{y} + \frac{3}{y^2} - 1 = 0$; $\frac{2}{-1} + \frac{3}{(-1)^2} - 1 \stackrel{?}{=} 0$; $-\frac{2}{1} + \frac{3}{1} - 1 \stackrel{?}{=} 0$; $\frac{-2 + 3}{1} - 1 \stackrel{?}{=} 0$; $1 - 1 \stackrel{?}{=} 0$; $0 = 0$

Thus, $\mathbf{y = 3}$ and $\mathbf{y = -1}$ are the real solutions to $\frac{2}{y} + \frac{3}{y^2} - 1 = 0$. In addition, the equation

$\frac{2}{y}+\frac{3}{y^2}-1=0$ can be factored to $-\boldsymbol{y}(\boldsymbol{y}-\mathbf{3})(\boldsymbol{y}+\mathbf{1})$.

Example 1.5-15

Solve the fractional equation $\frac{2y}{y-1}=\frac{1}{y^2-1}$.

Solution:

First - Write the denominator in the right hand side of the equation in its factored form.

$$\frac{2y}{y-1}=\frac{1}{y^2-1} \;;\; \frac{2y}{y-1}=\frac{1}{(y-1)(y+1)}$$

Second - Simplify the equation and cross multiply the terms in both sides of the equation.

$$\frac{2y}{y-1}=\frac{1}{(y-1)(y+1)} \;;\; \frac{2y}{1}=\frac{1}{(y+1)} \;;\; 2y(y+1)=1\cdot 1 \;;\; 2y^2+2y=1$$

Third - Write the quadratic equation in standard form, i.e., $2y^2+2y-1=0$

Fourth - Solve the quadratic equation by choosing a method.

$2y^2+2y-1=0 \;;\; \left(y+\frac{1-\sqrt{3}}{2}\right)\left(y+\frac{1+\sqrt{3}}{2}\right)=0$. Therefore, the two apparent solutions are:

$y+\frac{1-\sqrt{3}}{2}=0 \;;\; y+\frac{1-1.732}{2}=0 \;;\; y-0.37=0 \;;\; y=0.37$, and

$y+\frac{1+\sqrt{3}}{2}=0 \;;\; y+\frac{1+1.732}{2}=0 \;;\; y+1.37=0 \;;\; y=-1.37$

Fifth - Check the answers by substituting the y values into the original equation.

I. *Let* $y=0.37$ *in* $\frac{2y}{y-1}=\frac{1}{y^2-1} \;;\; \frac{2\times 0.37}{0.37-1}\stackrel{?}{=}\frac{1}{(0.37)^2-1} \;;\; \frac{0.74}{-0.63}\stackrel{?}{=}\frac{1}{0.137-1} \;;\; -1.17=-1.17$

II. *Let* $y=-1.37$ *in* $\frac{2y}{y-1}=\frac{1}{y^2-1} \;;\; \frac{2\times -1.37}{-1.37-1}\stackrel{?}{=}\frac{1}{(-1.37)^2-1} \;;\; \frac{-2.74}{-2.37}\stackrel{?}{=}\frac{1}{1.88-1} \;;\; 1.15=1.15$

Thus, $\boldsymbol{y}=\mathbf{0.37}$ and $\boldsymbol{y}=\mathbf{-1.37}$ are the real solutions to $\frac{2y}{y-1}=\frac{1}{y^2-1}$. In addition, the equation $\frac{2y}{y-1}=\frac{1}{y^2-1}$ can be factored to $(\boldsymbol{y}+\mathbf{1.37})(\boldsymbol{y}-\mathbf{0.37})=\mathbf{0}$.

Additional Examples - Solving Quadratic Equations Containing Fractions

The following examples further illustrate how to solve quadratic equations with fractional coefficients:

Example 1.5-16

Solve the fractional equation $x+5=\frac{-4}{x}$.

Solution:

First - Write the left hand side of the equation in fraction form. $x+5=\frac{-4}{x} \;;\; \frac{x+5}{1}=\frac{-4}{x}$

Second - Cross multiply the terms in both sides of the equation. $x\cdot(x+5)=1\cdot(-4) \;;\; x^2+5x=-4$

Third - Write the quadratic equation in standard form, i.e., $x^2+5x+4=0$

Fourth - Solve the quadratic equation by choosing a method. $x^2+5x+4=0$; $(x+1)(x+4)=0$.

Therefore, the two apparent solutions are: $x+1=0$; $x=-1$ and $x+4=0$; $x=-4$

Fifth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=-1$ *in* $x+5=\frac{-4}{x}$; $-1+5\overset{?}{=}\frac{-4}{-1}$; $4=4$

II. *Let* $x=-4$ *in* $x+5=\frac{-4}{x}$; $-4+5\overset{?}{=}\frac{-4}{-4}$; $1\overset{?}{=}\frac{1}{1}$; $1=1$

Therefore, $\boldsymbol{x=-1}$ and $\boldsymbol{x=-4}$ are the real solutions to $x+5=\frac{-4}{x}$. In addition, the equation $x+5=\frac{-4}{x}$ can be factored to $\boldsymbol{(x+1)(x+4)=0}$.

Example 1.5-17

Solve the fractional equation $6x+13=\frac{-5}{x}$.

Solution:

First - Write the left hand side of the equation in fraction form. $6x+13=\frac{-5}{x}$; $\frac{6x+13}{1}=\frac{-5}{x}$

Second - Cross multiply the terms in both sides of the equation.

$x\cdot(6x+13)=1\cdot(-5)$; $6x^2+13x=-5$

Third - Write the quadratic equation in standard form, i.e., $6x^2+13x+5=0$

Fourth - Solve the quadratic equation by choosing a method.

$6x^2+13x+5=0$; $(3x+5)(2x+1)=0$. Therefore, the two apparent solutions are:

$3x+5=0$; $3x=-5$; $x=-\frac{5}{3}$; $x=-1.67$ and $2x+1=0$; $2x=-1$; $x=-\frac{1}{2}$; $x=-0.5$

Fifth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=-1.67$ *in* $6x+13=\frac{-5}{x}$; $6\cdot(-1.67)+13\overset{?}{=}\frac{-5}{-1.67}$; $-10+13\overset{?}{=}3$; $3=3$

II. *Let* $x=-0.5$ *in* $6x+13=\frac{-5}{x}$; $6\cdot(-0.5)+13\overset{?}{=}\frac{-5}{-0.5}$; $-3+13\overset{?}{=}10$; $10=10$

Therefore, $\boldsymbol{x=-1.67}$ and $\boldsymbol{x=-0.5}$ are the real solutions to $6x+13=\frac{-5}{x}$. In addition, equation $6x+13=\frac{-5}{x}$ can be factored to $\boldsymbol{(3x+5)(2x+1)=0}$.

Example 1.5-18

Solve the fractional equation $y=\frac{25}{y}$.

Solution:

First - Write the left hand side of the equation in fraction form. $y=\frac{25}{y}$; $\frac{y}{1}=\frac{25}{y}$

Second - Cross multiply the terms in both sides of the equation. $y\cdot y=1\cdot 25$; $y^2=25$

Third - Solve the quadratic equation by choosing the Square Root method.

$y^2=25$; $\sqrt{y^2}=\pm\sqrt{25}$; $y=\pm\sqrt{5^2}$; $y=\pm 5$. Therefore, the two apparent solutions are:

$y=+5$ and $y=-5$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $y=5$ *in* $y=\frac{25}{y}$; $5\overset{?}{=}\frac{\overset{5}{\cancel{25}}}{\cancel{5}}$; $5\overset{?}{=}\frac{5}{1}$; $5=5$

II. *Let* $y=-5$ *in* $y=\frac{25}{y}$; $-5\overset{?}{=}\frac{25}{-5}$; $-5\overset{?}{=}-\frac{\overset{5}{\cancel{25}}}{\cancel{5}}$; $-5\overset{?}{=}-\frac{5}{1}$; $-5=-5$

Therefore, $\mathbf{y=5}$ and $\mathbf{y=-5}$ are the real solutions to $y=\frac{25}{y}$. In addition, the equation $y=\frac{25}{y}$ can be factored to $\mathbf{(y-5)(y+5)=0}$.

Example 1.5-19

Solve the fractional equation $\frac{2y-15}{y}+y=0$.

Solution:

First - use fraction techniques to rewrite the equation in quadratic form.

$$\frac{2y-15}{y}+y=0\ ;\ \frac{2y-15}{y}+\frac{y}{1}=0\ ;\ \frac{[1\cdot(2y-15)]+(y\cdot y)}{1\cdot y}=0\ ;\ \frac{2y-15+y^2}{y}=0\ ;\ \frac{2y-15+y^2}{y}=\frac{0}{1}$$
$$;\ 1\cdot\left(2y-15+y^2\right)=y\cdot 0\ ;\ 2y-15+y^2=0$$

Second - write the quadratic equation in standard form, i.e., $y^2+2y-15=0$

Third - Solve the quadratic equation by choosing a method.

$$y^2+2y-15=0\ ;\ (y+5)(y-3)=0.$$

Therefore, the two apparent solutions are: $y+5=0$; $y=-5$ and $y-3=0$; $y=3$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $y=-5$ *in* $\frac{2y-15}{y}+y=0$; $\frac{2\cdot(-5)-15}{-5}-5\overset{?}{=}0$; $\frac{-10-15}{-5}-5\overset{?}{=}0$; $\frac{-25}{-5}-5\overset{?}{=}0$; $0=0$

II. *Let* $y=3$ *in* $\frac{2y-15}{y}+y=0$; $\frac{6-15}{3}+3\overset{?}{=}0$; $\frac{-9}{3}+3\overset{?}{=}0$; $-3+3\overset{?}{=}0$; $0=0$

Therefore, $\mathbf{y=-5}$ and $\mathbf{y=3}$ are the real solutions to $\frac{2y-15}{y}+y=0$. In addition, the equation $\frac{2y-15}{y}+y=0$ can be factored to $\mathbf{(y+5)(y-3)=0}$.

Example 1.5-20

Solve the fractional equation $\frac{x^2}{x+1}=\frac{4}{x+1}$.

Solution:

First - Cross multiply the terms in both sides of the equation.

$$\frac{x^2}{x+1}=\frac{4}{x+1}\ ;\ x^2\cdot(x+1)=4\cdot(x+1).$$

Note that $x+1$ can be eliminated from both sides of the equation where we obtain $x^2=4$.

Second - Factor out the quadratic equation by choosing the Square Root factoring method.

$$x^2=4\ ;\ \sqrt{x^2}=\pm\sqrt{4}\ ;\ x=\pm\sqrt{2^2}\ ;\ x=\pm 2.$$

Therefore, the two apparent solutions are: $x=+2$ and $x=-2$

Third - Check the answers by substituting the x values into the original equation.

I. *Let* $x = 2$ *in* $\frac{x^2}{x+1} = \frac{4}{x+1}$; $\frac{2^2}{2+1} \stackrel{?}{=} \frac{4}{2+1}$; $\frac{4}{3} = \frac{4}{3}$

II. *Let* $x = -2$ *in* $\frac{x^2}{x+1} = \frac{4}{x+1}$; $\frac{(-2)^2}{-2+1} \stackrel{?}{=} \frac{4}{-2+1}$; $\frac{4}{-1} = \frac{4}{-1}$; $-4 = -4$

Therefore, $\boldsymbol{x = 2}$ and $\boldsymbol{x = -2}$ are the real solutions to $\frac{x^2}{x+1} = \frac{4}{x+1}$. In addition, the equation $\frac{x^2}{x+1} = \frac{4}{x+1}$ can be factored to $\boldsymbol{(x+2)(x-2) = 0}$.

Practice Problems - Solving Quadratic Equations Containing Fractions

Section 1.5 Case II Practice Problems - Solve the following equations. Check the answers by substituting the solution into the original equation.

1. $\frac{8}{y+1} = y - 1$
2. $\frac{11x+15}{x} = -2x$
3. $\frac{x^2}{x+3} = \frac{1}{x+3}$
4. $\frac{1-2u}{u} = -u$
5. $x = \frac{3}{x} - 2$
6. $\frac{3x-10}{x} = -x$
7. $u = \frac{49}{u}$
8. $6x + 17 = \frac{-5}{x}$
9. $y + 4 = -\frac{3}{y}$
10. $3x = \frac{-5x-2}{x}$

1.6 How to Choose the Best Factoring or Solution Method

To factor polynomials and to solve quadratic equations a total of seven basic methods have been introduced in Chapter 3 of the Mastering Algebra – Intermediate Level book and Sections 1.2–1.4 of this manual. Those methods are:

1. The Greatest Common Factoring method
2. The Grouping method
3. The Trial and Error method
4. Factoring methods for polynomials with square and cubed terms
5. The Quadratic Formula method
6. The Square Root Property method, and
7. Completing-the-Square method

The decision as to which one of the above methods is most suitable in factoring a polynomial or solving an equation is left to the student. For example, in some cases, using the Trial and Error method in solving a quadratic equation may be easier than using the Quadratic Formula or Completing-the-Square method. In certain cases, using the quadratic formula in solving a polynomial may be faster than the Grouping or the Trial and Error method. Note that the key in choosing the best and/or the easiest method is through solving many problems. After sufficient practice, students start to gain confidence on selection of one method over the other.

Assumption - *In many instances, the methods used in factoring polynomials (shown in Chapter 3 of the Mastering Algebra – Intermediate Level book) can also be used in solving quadratic equations (shown in Sections 1.2–1.5 of this manual) by recognizing that the left hand side of the equation* $ax^2 + bx + c = 0$*, namely* $ax^2 + bx + c$ *is a polynomial and can be factored as such, using polynomial factoring methods covered in Chapter 3 of the Mastering Algebra – Intermediate Level book.*

Note 1 - Any quadratic equation can be solved using the quadratic formula. Once the student has memorized the quadratic formula and has learned how to substitute the equivalent values of a, b, and c into the quadratic formula, then the next steps are merely the process of solving the quadratic equation using mathematical operations.

Note 2 - The quadratic formula can be used as an alternative method in factoring polynomials of the form $ax^2 + bx + c$ as is stated in the above assumption.

The following examples are solved using the seven factoring and solution methods shown above:

Example 1.6-1

Use different methods to solve the equation $x^2 = 25$.

Solution:

First Method: (The Trial and Error Method)

Write the equation in the standard quadratic equation form $ax^2 + bx + c = 0$, i.e., write $x^2 = 25$ as $x^2 + 0x - 25 = 0$. To solve the given equation using the Trial and Error method we only consider the left hand side of the equation which is a second degree polynomial. Next, we need to obtain two numbers whose sum is 0 and whose product is -25 by constructing a table as shown below:

Sum	***Product***
$1-1=0$	$1\cdot(-1)=-1$
$2-2=0$	$2\cdot(-2)=-4$
$3-3=0$	$3\cdot(-3)=-9$
$4-4=0$	$4\cdot(-4)=-16$
$\mathbf{5-5=0}$	$\mathbf{5\cdot(-5)=-25}$

The last line contains the sum and the product of the two numbers that we need. Thus, $x^2 = 25$ or $x^2 + 0x - 25 = 0$ can be factored to $\boldsymbol{(x-5)(x+5)=0}$.

Check: $(x-5)(x+5)=0$; $x\cdot x + 5\cdot x - 5\cdot x + 5\cdot(-5) = 0$; $x^2 + 5x - 5x - 25 = 0$; $x^2 + (5-5)x - 25 = 0$; $x^2 + 0x - 25 = 0$

Second Method: (The Quadratic Formula Method)

First, write the equation in the standard quadratic equation form $ax^2 + bx + c = 0$, i.e., write $x^2 = 25$ as $x^2 + 0x - 25 = 0$. Second, equate the coefficients of $x^2 + 0x - 25 = 0$ with the standard quadratic equation by letting $a = 1$, $b = 0$, and $c = -25$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-0 \pm \sqrt{0^2 - (4 \times 1 \times -25)}}{2 \times 1}$; $x = \frac{\pm\sqrt{0+100}}{2}$; $x = \frac{\pm\sqrt{100}}{2}$; $x = \pm\frac{\sqrt{10^2}}{2}$; $x = \pm\frac{10}{2}$. Therefore:

I. $x = +\frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}}$; $x = \frac{5}{1}$; $\boldsymbol{x = 5}$ II. $x = -\frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}}$; $x = -\frac{5}{1}$; $\boldsymbol{x = -5}$

Check: I. *Let* $x = 5$ *in* $x^2 = 25$; $5^2 \overset{?}{=} 25$; $25 = 25$

II. *Let* $x = -5$ *in* $x^2 = 25$; $(-5)^2 \overset{?}{=} 25$; $25 = 25$

Therefore, the equation $x^2 + 0x - 25 = 0$ can be factored to $\boldsymbol{(x+5)(x-5)=0}$.

Third Method: (The Square Root Property Method)

Take the square root of both sides of the equation, i.e., write $x^2 = 25$ as $\sqrt{x^2} = \pm\sqrt{25}$; $x = \pm\sqrt{5^2}$; $x = \pm 5$. Thus, $\boldsymbol{x = +5}$ or $\boldsymbol{x = -5}$ are the solution sets to the equation $x^2 = 25$ which can be represented in its factorable form as $\boldsymbol{(x+5)(x-5)=0}$.

Fourth Method: (Completing-the-Square Method) - Is not applicable.

Note that from the above three methods using the Square Root Property method is the fastest and the easiest method to obtain the factored terms. The Trial and Error method is the second easiest method to use, followed by the Quadratic Formula method which is the most difficult way of obtaining the factored terms.

Example 1.6-2

Use different methods to solve the equation $x^2 + 11x + 24 = 0$.

Solution:

First Method: (The Trial and Error Method)

To solve the given equation using the Trial and Error method we only consider the left hand

side of the equation which is a second degree polynomial. Next, we need to obtain two numbers whose sum is 11 and whose product is 24 by constructing a table as shown below:

Sum	***Product***
$6+5=11$	$6\cdot 5=30$
$7+4=11$	$7\cdot 4=28$
$\mathbf{8+3=11}$	$\mathbf{8\cdot 3=24}$
$9+2=11$	$9\cdot 2=18$

The third line contains the sum and the product of the two numbers that we need. Thus, $x^2+11x+24=0$ can be factored to $\mathbf{(x+8)(x+3)=0}$.

Check: $(x+8)(x+3)=0$; $x\cdot x+3\cdot x+8\cdot x+8\cdot 3=0$; $x^2+3x+8x+24=0$; $x^2+(3+8)x+24=0$

; $x^2+11x+24=0$

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2+bx+c=0$, equate the coefficients of $x^2+11x+24=0$ with the standard quadratic equation by letting $a=1$, $b=11$, and $c=24$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-11\pm\sqrt{11^2-(4\times 1\times 24)}}{2\times 1}$; $x=\frac{-11\pm\sqrt{121-96}}{2}$; $x=\frac{-11\pm\sqrt{25}}{2}$

; $x=\frac{-11\pm\sqrt{5^2}}{2}$; $x=\frac{-11\pm 5}{2}$. Therefore:

I. $x=\frac{-11+5}{2}$; $x=-\frac{\overset{3}{\cancel{6}}}{\cancel{2}}$; $x=-\frac{3}{1}$; $\boldsymbol{x=-3}$

II. $x=\frac{-11-5}{2}$; $x=-\frac{\overset{8}{\cancel{16}}}{\cancel{2}}$; $x=-\frac{8}{1}$; $\boldsymbol{x=-8}$

Check: I. *Let* $x=-3$ *in* $x^2+11x+24=0$; $(-3)^2+11\cdot(-3)+24\overset{?}{=}0$; $9-33+24\overset{?}{=}0$; $33-33\overset{?}{=}0$

; $0=0$

II. *Let* $x=-8$ *in* $x^2+11x+24=0$; $(-8)^2+11\cdot(-8)+24\overset{?}{=}0$; $64-88+24\overset{?}{=}0$; $88-88\overset{?}{=}0$

; $0=0$

Therefore, the equation $x^2+11x+24=0$ can be factored to $\mathbf{(x+8)(x+3)=0}$.

Third Method: (Completing-the-Square Method)

$x^2+11x+24=0$; $x^2+11x=-24$; $x^2+11x+\left(\frac{11}{2}\right)^2=-24+\left(\frac{11}{2}\right)^2$; $x^2+11x+\frac{121}{4}=-24+\frac{121}{4}$

; $\left(x+\frac{11}{2}\right)^2=-\frac{24}{1}+\frac{121}{4}$; $\left(x+\frac{11}{2}\right)^2=\frac{(-24\cdot 4)+(1\cdot 121)}{1\cdot 4}$; $\left(x+\frac{11}{2}\right)^2=\frac{-96+121}{4}$; $\left(x+\frac{11}{2}\right)^2=\frac{25}{4}$

; $x+\frac{11}{2}=\pm\sqrt{\frac{25}{4}}$; $x+\frac{11}{2}=\pm\frac{5}{2}$

Therefore: I. $x+\frac{11}{2}=+\frac{5}{2}$; $x=\frac{5}{2}-\frac{11}{2}$; $x=\frac{5-11}{2}$; $x=-\frac{\overset{3}{\cancel{6}}}{\cancel{2}}$; $x=-\frac{3}{1}$; $\boldsymbol{x=-3}$

II. $x+\frac{11}{2}=-\frac{5}{2}$; $x=-\frac{5}{2}-\frac{11}{2}$; $x=\frac{-5-11}{2}$; $x=-\frac{\overset{8}{\cancel{16}}}{\cancel{2}}$; $x=-\frac{8}{1}$; $\boldsymbol{x=-8}$

Check: I. *Let* $x=-3$ *in* $x^2+11x+24=0$; $(-3)^2+(11\times -3)+24\overset{?}{=}0$; $9-33+24\overset{?}{=}0$; $0=0$

II. *Let* $x=-8$ *in* $x^2+11x+24=0$; $(-8)^2+(11\times -8)+24\overset{?}{=}0$; $64-88+24\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2 + 11x + 24 = 0$ can be factored to $\mathbf{(x+8)(x+3)=0}$.

Fourth Method: (The Square Root Property Method) - Is not applicable

Note that from the above three methods using the Trial and Error method is the fastest and the easiest method to obtain the factored terms. Completing-the-Square method is the second easiest method to use, followed by the Quadratic Formula method which is the longest and most difficult way of obtaining the factored terms.

Example 1.6-3

Use different methods to solve the equation $x^2 + 5x + 2 = 0$.

Solution:

First Method: (The Trial and Error Method)

To solve the given equation using the Trial and Error method we only consider the left hand side of the equation which is a second degree polynomial. Next, we need to obtain two numbers whose sum is 5 and whose product is 2. However, after few trials, it becomes clear that such a combination of integer numbers is not possible to obtain. Hence, the Trial and Error method is not applicable to this particular example.

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2 + bx + c = 0$, equate the coefficients of $x^2 + 5x + 2 = 0$ with the standard quadratic equation by letting $a = 1$, $b = 5$, and $c = 2$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-5 \pm \sqrt{5^2 - (4 \times 1 \times 2)}}{2 \times 1}$; $x = \frac{-5 \pm \sqrt{25-8}}{2}$; $x = \frac{-5 \pm \sqrt{17}}{2}$. Therefore:

I. $x = \frac{-5+\sqrt{17}}{2}$; $x = \frac{-5+4.12}{2}$; $x = -\frac{0.88}{2}$; $\mathbf{x = -0.44}$

II. $x = \frac{-5-\sqrt{17}}{2}$; $x = \frac{-5-4.12}{2}$; $x = -\frac{9.12}{2}$; $\mathbf{x = -4.56}$

Check: I. *Let* $x = -0.44$ *in* $x^2 + 5x + 2 = 0$; $(-0.44)^2 + 5 \cdot (-0.44) + 2 \stackrel{?}{=} 0$; $0.2 - 2.2 + 2 \stackrel{?}{=} 0$; $2.2 - 2.2 \stackrel{?}{=} 0$; $0 = 0$

II. *Let* $x = -4.56$ *in* $x^2 + 5x + 2 = 0$; $(-4.56)^2 + 5 \cdot (-4.56) + 2 \stackrel{?}{=} 0$; $20.8 - 22.8 + 2 \stackrel{?}{=} 0$; $22.8 - 22.8 \stackrel{?}{=} 0$; $0 = 0$

Therefore, the equation $x^2 + 5x + 2 = 0$ can be factored to $\mathbf{(x+0.44)(x+4.56)=0}$.

Third Method: (Completing-the-Square Method)

$x^2 + 5x + 2 = 0$; $x^2 + 5x = -2$; $x^2 + 5x + \left(\frac{5}{2}\right)^2 = -2 + \left(\frac{5}{2}\right)^2$; $x^2 + 5x + \frac{25}{4} = -2 + \frac{25}{4}$; $\left(x + \frac{5}{2}\right)^2 = -\frac{2}{1} + \frac{25}{4}$; $\left(x + \frac{5}{2}\right)^2 = \frac{(-2 \cdot 4) + (1 \cdot 25)}{1 \cdot 4}$; $\left(x + \frac{5}{2}\right)^2 = \frac{-8+25}{4}$; $\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$; $x + \frac{5}{2} = \pm\sqrt{\frac{17}{4}}$; $x + \frac{5}{2} = \pm\frac{\sqrt{17}}{2}$

Therefore: I. $x + \frac{5}{2} = +\frac{\sqrt{17}}{2}$; $x = \frac{\sqrt{17}}{2} - \frac{5}{2}$; $x = \frac{\sqrt{17}-5}{2}$; $x = \frac{4.12-5}{2}$; $x = -\frac{0.88}{2}$; $\mathbf{x = -0.44}$

II. $x + \frac{5}{2} = -\frac{\sqrt{17}}{2}$; $x = -\frac{\sqrt{17}}{2} - \frac{5}{2}$; $x = \frac{-\sqrt{17}-5}{2}$; $x = \frac{-4.12-5}{2}$; $x = -\frac{9.12}{2}$; $\mathbf{x = -4.56}$

Check: I. $Let\ x = -0.44\ \ in\quad x^2 + 5x + 2 = 0$; $(-0.44)^2 + 5\cdot(-0.44) + 2 \stackrel{?}{=} 0$; $0.2 - 2.2 + 2 \stackrel{?}{=} 0$; $2.2 - 2.2 \stackrel{?}{=} 0$; $0 = 0$

II. $Let\ x = -4.56\ \ in\quad x^2 + 5x + 2 = 0$; $(-4.56)^2 + 5\cdot(-4.56) + 2 \stackrel{?}{=} 0$; $20.8 - 22.8 + 2 \stackrel{?}{=} 0$; $22.8 - 22.8 \stackrel{?}{=} 0$; $0 = 0$

Therefore, the equation $x^2 + 5x + 2 = 0$ can be factored to $\mathbf{(x + 0.44)(x + 4.56) = 0}$.

Fourth Method: (The Square Root Property Method) - Is not applicable.

Note that from the above two methods using the Quadratic Formula method may be the faster method, for some, than Completing-the-Square method.

Example 1.6-4

Use different methods to solve the equation $6x^2 + 4x - 2 = 0$.

Solution:

First Divide both sides of the equation by 2, i.e., $6x^2 + 4x - 2 = 0$; $\frac{\overset{3}{\cancel{6}}}{\cancel{2}}x^2 + \frac{\overset{2}{\cancel{4}}}{\cancel{2}}x - \frac{\cancel{2}}{\cancel{2}} = 0$;

$3x^2 + 2x - 1 = 0$. Then consider other methods to solve the equation $3x^2 + 2x - 1 = 0$.

First Method: (The Trial and Error Method)

To solve the given equation using the Trial and Error method we only consider the left hand side of the equation which is a second degree polynomial. Next, we need to obtain two numbers whose sum is 2 and whose product is $3\cdot -1 = -3$ by constructing a table as shown below:

Sum	***Product***
$6 - 4 = 2$	$6\cdot(-4) = -24$
$5 - 3 = 2$	$5\cdot(-3) = -15$
$4 - 2 = 2$	$4\cdot(-2) = -8$
$\mathbf{3 - 1 = 2}$	$\mathbf{3\cdot(-1) = -3}$

The last line contains the sum and the product of the two numbers that we need. Therefore,

$3x^2 + 2x - 1 = 0$; $3x^2 + (3-1)x - 1 = 0$; $3x^2 + 3x - x - 1 = 0$; $3x(x+1) - (x+1) = 0$; $\mathbf{(x+1)(3x-1) = 0}$.

Check: $(x+1)(3x-1) = 0$; $3\cdot x\cdot x - 1\cdot x + (1\cdot 3)\cdot x + 1\cdot(-1) = 0$; $3x^2 - x + 3x - 1 = 0$; $3x^2 + (3-1)x - 1 = 0$; $3x^2 + 2x - 1 = 0$

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2 + bx + c = 0$, equate the coefficients of $3x^2 + 2x - 1 = 0$ with the standard quadratic equation by letting $a = 3$, $b = 2$, and $c = -1$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-2 \pm \sqrt{2^2 - (4\times 3\times -1)}}{2\times 3}$; $x = \frac{-2 \pm \sqrt{4+12}}{6}$; $x = \frac{-2 \pm \sqrt{16}}{6}$; $x = \frac{-2 \pm \sqrt{4^2}}{6}$; $x = \frac{-2 \pm 4}{6}$. Therefore:

I. $x = \frac{-2+4}{6}$; $x = \frac{\cancel{2}}{\underset{3}{\cancel{6}}}$; $\mathbf{x = \frac{1}{3}}$

II. $x = \frac{-2-4}{6}$; $x = -\frac{\cancel{6}}{\cancel{6}}$; $x = -\frac{1}{1}$; $\mathbf{x = -1}$

Check: I. $Let\ x=\frac{1}{3}\ in\quad 3x^2+2x-1=0\ ;\ 3\cdot\left(\frac{1}{3}\right)^2+2\cdot\left(\frac{1}{3}\right)-1\stackrel{?}{=}0\ ;\ \frac{\cancel{3}}{\frac{\cancel{9}}{3}}+\frac{2}{3}-1\stackrel{?}{=}0\ ;\ \frac{1}{3}+\frac{2}{3}-1\stackrel{?}{=}0$

$;\ \frac{1+2}{3}-1\stackrel{?}{=}0\ ;\ \frac{\cancel{3}}{\cancel{3}}-1\stackrel{?}{=}0\ ;\ 1-1\stackrel{?}{=}0\ ;\ 0=0$

II. $Let\ x=-1\ in\quad 3x^2+2x-1=0\ ;\ 3\cdot(-1)^2+2\cdot(-1)-1\stackrel{?}{=}0\ ;\ 3-2-1\stackrel{?}{=}0\ ;\ 3-3\stackrel{?}{=}0\ ;\ 0=0$

Therefore, the equation $3x^2+2x-1=0$ can be factored to $\left(x-\frac{1}{3}\right)(x+1)=0$ which is the same as $\mathbf{(3x-1)(x+1)=0}$.

Third Method: (Completing-the-Square Method)

$3x^2+2x-1=0\ ;\ 3x^2+2x=1\ ;\ \frac{\cancel{3}}{\cancel{3}}x^2+\frac{2}{3}x=\frac{1}{3}\ ;\ x^2+\frac{2}{3}x=\frac{1}{3}\ ;\ x^2+\frac{2}{3}x+\left(\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\right)^2=\frac{1}{3}+\left(\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\right)^2$

$;\ x^2+\frac{2}{3}x+\frac{1}{9}=\frac{1}{3}+\frac{1}{9}\ ;\ \left(x+\frac{1}{3}\right)^2=\frac{(1\cdot 9)+(1\cdot 3)}{3\cdot 9}\ ;\ \left(x+\frac{1}{3}\right)^2=\frac{9+3}{27}\ ;\ \left(x+\frac{1}{3}\right)^2=\frac{12}{27}\ ;\ x+\frac{1}{3}=\pm\sqrt{\frac{12}{27}}$

$;\ x+\frac{1}{3}=\pm\sqrt{\frac{4\cdot\cancel{3}}{9\cdot\cancel{3}}}\ ;\ x+\frac{1}{3}=\pm\sqrt{\frac{2^2}{3^2}}\ ;\ x+\frac{1}{3}=\pm\frac{2}{3}$

Therefore: I. $x+\frac{1}{3}=+\frac{2}{3}\ ;\ x=\frac{2}{3}-\frac{1}{3}\ ;\ x=\frac{2-1}{3}\ ;\ \boldsymbol{x=\frac{1}{3}}$

II. $x+\frac{1}{3}=-\frac{2}{3}\ ;\ x=-\frac{1}{3}-\frac{2}{3}\ ;\ x=\frac{-1-2}{3}\ ;\ x=-\frac{\cancel{3}}{\cancel{3}}\ ;\ x=-\frac{1}{1}\ ;\ \boldsymbol{x=-1}$

Check: I. $Let\ x=\frac{1}{3}\ in\quad 3x^2+2x-1=0\ ;\ 3\cdot\left(\frac{1}{3}\right)^2+2\cdot\left(\frac{1}{3}\right)-1\stackrel{?}{=}0\ ;\ \frac{\cancel{3}}{\frac{\cancel{9}}{3}}+\frac{2}{3}-1\stackrel{?}{=}0\ ;\ \frac{1}{3}+\frac{2}{3}-1\stackrel{?}{=}0$

$;\ \frac{1+2}{3}-1\stackrel{?}{=}0\ ;\ \frac{\cancel{3}}{\cancel{3}}-1\stackrel{?}{=}0\ ;\ 1-1\stackrel{?}{=}0\ ;\ 0=0$

II. $Let\ x=-1\ in\quad 3x^2+2x-1=0\ ;\ 3\cdot(-1)^2+2\cdot(-1)-1\stackrel{?}{=}0\ ;\ 3-2-1\stackrel{?}{=}0\ ;\ 3-3\stackrel{?}{=}0\ ;\ 0=0$

Therefore, the equation $3x^2+2x-1=0$ can be factored to $\left(x-\frac{1}{3}\right)(x+1)=0$ which is the same as $\mathbf{(3x-1)(x+1)=0}$.

Fourth Method: (The Square Root Property Method) - Is not applicable.

Note that from the above three methods using the Trial and Error method is the easiest method to obtain the factored terms. The Quadratic Formula method is the second easiest method to use, followed by Completing-the-Square method.

Example 1.6-5

Use different methods to solve the equation $(2x+5)^2=25$.

Solution:

First Method: (The Trial and Error Method)

To apply the Trial and Error method to the equation $(2x+5)^2=25$ we need to complete and simplify the square in the left hand side of the equation, i.e., $(2x+5)^2=25\ ;\ 4x^2+25+20x=25$

; $4x^2+20x+25-25=25-25$; $4x^2+20x+0=0$; $\frac{\cancel{4}}{\cancel{4}}x^2+\frac{\overset{5}{\cancel{20}}}{\cancel{4}}x+0=0$; $x^2+5x+0=0$. Then, we only consider the left hand side of the equation which is a second degree polynomial. Next we need to obtain two numbers whose sum is 5 and whose product is $5\cdot 0=0$ by constructing a table as shown below:

Sum	***Product***
$3+2=5$	$3\cdot 2=6$
$4+1=5$	$4\cdot 1=4$
$\mathbf{5+0=5}$	$\mathbf{5\cdot 0=0}$

The last line contains the sum and the product of the two numbers that we need. Thus, $(2x+5)^2=25$ can be factored to $(x+0)(x+5)=0$ which is the same as $\boldsymbol{x(x+5)=0}$

Second Method: (The Square Root Property Method)

$(2x+5)^2=25$; $\sqrt{(2x+5)^2}=\pm\sqrt{25}$; $2x+5=\pm 5$. Therefore:

I. $2x+5=+5$; $2x=5-5$; $2x=0$; $\boldsymbol{x=0}$

II. $2x+5=-5$; $2x=-5-5$; $2x=-10$; $x=-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $\boldsymbol{x=-5}$

Check: I. *Let* $x=0$ *in* $(2x+5)^2=25$; $[(2\cdot 0)+5]^2\overset{?}{=}25$; $5^2\overset{?}{=}25$; $25=25$

II. *Let* $x=-5$ *in* $(2x+5)^2=25$; $[(2\cdot -5)+5]^2\overset{?}{=}25$; $(-10+5)^2\overset{?}{=}25$; $(-5)^2\overset{?}{=}25$; $25=25$

Therefore, the equation $(2x+5)^2=25$ can be factored to $(x-0)(x+5)=0$ which is the same as $\boldsymbol{x(x+5)=0}$.

Third Method: (The Quadratic Formula Method)

First complete the square term on the left hand side and simplify the equation:

$(2x+5)^2=25$; $4x^2+20x+25=25$; $4x^2+20x=25-25$; $4x^2+20x=0$; $\frac{\cancel{4}}{\cancel{4}}x^2+\frac{\overset{5}{\cancel{20}}}{\cancel{4}}x=\frac{0}{4}$; $x^2+5x=0$

Then, given the standard quadratic equation $ax^2+bx+c=0$, equate the coefficients of $x^2+5x=0$ with the standard quadratic equation by letting $a=1$, $b=5$, and $c=0$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-5\pm\sqrt{5^2-(4\times 1\times 0)}}{2\times 1}$; $x=\frac{-5\pm\sqrt{25-0}}{2}$; $x=\frac{-5\pm 5}{2}$. Therefore:

I. $x=\frac{-5+5}{2}$; $x=\frac{0}{2}$; $\boldsymbol{x=0}$

II. $x=\frac{-5-5}{2}$; $x=-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $\boldsymbol{x=-5}$

Check: I. *Let* $x=0$ *in* $x^2+5x=0$; $0^2+5\times 0\overset{?}{=}0$; $0+0\overset{?}{=}0$; $0=0$

II. *Let* $x=-5$ *in* $x^2+5x=0$; $(-5)^2+5\cdot(-5)\overset{?}{=}0$; $25-25\overset{?}{=}0$; $0=0$

Therefore, the equation $(2x+5)^2=25$ can be factored to $(x-0)(x+5)=0$ which is the same as $\boldsymbol{x(x+5)=0}$.

Fourth Method: (The Greatest Common Factoring Method)

First complete the square term on the left hand side and simplify the equation:

$(2x+5)^2 = 25$; $4x^2 + 20x + 25 = 25$; $4x^2 + 20x = 25 - 25$; $4x^2 + 20x = 0$; $\frac{\cancel{4}}{\cancel{4}}x^2 + \frac{\overset{5}{\cancel{20}}}{\cancel{4}}x = \frac{0}{4}$; $x^2 + 5x = 0$

Then, Factor out the greatest common monomial term x, i.e., $x^2 + 5x = 0$; $x(x+5) = 0$. Thus, the two solution to the equation are:

I. $x = 0$ and II. $x + 5 = 0$; $\mathbf{x = -5}$

Hence, the equation $(2x+5)^2 = 25$ can be factored to $(x-0)(x+5) = 0$ which is the same as $\mathbf{x(x+5) = 0}$.

Fifth Method: (Completing-the-Square Method)

First complete the square term on the left hand side and simplify the equation:

$(2x+5)^2 = 25$; $4x^2 + 20x + 25 = 25$; $4x^2 + 20x = 25 - 25$; $4x^2 + 20x = 0$; $\frac{\cancel{4}}{\cancel{4}}x^2 + \frac{\overset{5}{\cancel{20}}}{\cancel{4}}x = \frac{0}{4}$; $x^2 + 5x = 0$

Then, complete the square.

$x^2 + 5x = 0$; $x^2 + 5x + \left(\frac{5}{2}\right)^2 = 0 + \left(\frac{5}{2}\right)^2$; $x^2 + 5x + \frac{25}{4} = \frac{25}{4}$; $\left(x + \frac{5}{2}\right)^2 = \frac{25}{4}$; $x + \frac{5}{2} = \pm\sqrt{\frac{25}{4}}$; $x + \frac{5}{2} = \pm\frac{5}{2}$.

Therefore: I. $x + \frac{5}{2} = +\frac{5}{2}$; $x = \frac{5}{2} - \frac{5}{2}$; $x = \frac{5-5}{2}$; $x = \frac{0}{2}$; $\mathbf{x = 0}$

II. $x + \frac{5}{2} = -\frac{5}{2}$; $x = -\frac{5}{2} - \frac{5}{2}$; $x = \frac{-5-5}{2}$; $x = -\frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $\mathbf{x = -5}$

Check: I. *Let* $x = 0$ *in* $x^2 + 5x = 0$; $0^2 + 5 \times 0 \overset{?}{=} 0$; $0 + 0 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = -5$ *in* $x^2 + 5x = 0$; $(-5)^2 + 5 \cdot (-5) \overset{?}{=} 0$; $25 - 25 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $(2x+5)^2 = 25$ can be factored to $(x-0)(x+5) = 0$ which is the same as $\mathbf{x(x+5) = 0}$.

Note that from the above five methods the Square Root Property and the Trial and Error methods are the easiest methods in solving the quadratic equation, followed by the Greatest Common Factoring method, Quadratic Formula method, and Completing-the-Square method.

Practice Problems - How to Choose the Best Factoring or Solution Method

Section 1.6 Practice Problems - Choose three methods to solve the following quadratic equations. State the degree of difficulty associated with each method you selected.

1. $x^2 = 16$
2. $x^2 + 7x + 3 = 0$
3. $(3x+4)^2 = 36$
4. $x^2 + 11x + 30 = 0$
5. $5t^2 + 4t - 1 = 0$
6. $(2x+6)^2 = 36$
7. $y^2 - 8y + 15 = 0$
8. $w^2 = -7$
9. $6x^2 + x - 1 = 0$
10. $x^2 - 4x + 4$

Appendix – Exercise Solutions

Section 1.1 Solutions - Quadratic Equations and the Quadratic Formula

1. First - Write the quadratic equation $3x = -5 + 2x^2$ in standard form $ax^2 + bx + c = 0$.

   $3x = -5 + 2x^2$; $-2x^2 + 3x = -5 + 2x^2 - 2x^2$; $-2x^2 + 3x = -5 + 0$; $-2x^2 + 3x = -5$; $-2x^2 + 3x + 5 = -5 + 5$; $-2x^2 + 3x + 5 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = -2}$, $\boldsymbol{b = 3}$, and $\boldsymbol{c = 5}$

2. First - Write the quadratic equation $2x^2 = 5$ in standard form $ax^2 + bx + c = 0$.

   $2x^2 = 5$; $2x^2 - 5 = 5 - 5$; $2x^2 - 5 = 0$ which is the same as $2x^2 + 0x - 5 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = 2}$, $\boldsymbol{b = 0}$, and $\boldsymbol{c = -5}$

3. First - Write the quadratic equation $3w^2 - 5w = 2$ in standard form $aw^2 + bw + c = 0$.

   $3w^2 - 5w = 2$; $3w^2 - 5w - 2 = 2 - 2$; $3w^2 - 5w - 2 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = 3}$, $\boldsymbol{b = -5}$, and $\boldsymbol{c = -2}$

4. First - Write the quadratic equation $15 = -y^2 - 3$ in standard form $ay^2 + by + c = 0$.

   $15 = -y^2 - 3$; $y^2 + 15 = -y^2 + y^2 - 3$; $y^2 + 15 = 0 - 3$; $y^2 + 15 = -3$; $y^2 + 15 + 3 = -3 + 3$; $y^2 + 18 = 0$

   ; which is the same as $y^2 + 0y + 18 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = 1}$, $\boldsymbol{b = 0}$, and $\boldsymbol{c = 18}$

5. First - Write the quadratic equation $x^2 + 3 = 5x$ in standard form $ax^2 + bx + c = 0$.

   $x^2 + 3 = 5x$; $x^2 - 5x + 3 = 5x - 5x$; $x^2 - 5x + 3 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = 1}$, $\boldsymbol{b = -5}$, and $\boldsymbol{c = 3}$

6. First - Write the quadratic equation $-u^2 + 2 = 3u$ in standard form $au^2 + bu + c = 0$.

   $-u^2 + 2 = 3u$; $-u^2 - 3u + 2 = 3u - 3u$; $-u^2 - 3u + 2 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = -1}$, $\boldsymbol{b = -3}$, and $\boldsymbol{c = 2}$

7. The quadratic equation $y^2 + 5y - 2 = 0$ is already in standard form $ay^2 + by + c = 0$. Therefore, simply equate the a, b, and c coefficients with the coefficients of the given quadratic equation to obtain $\boldsymbol{a = 1}$, $\boldsymbol{b = 5}$, and $\boldsymbol{c = -2}$.

8. First - Write the quadratic equation $-3x^2 = 2x - 1$ in standard form $ax^2 + bx + c = 0$.

   $-3x^2 = 2x - 1$; $-3x^2 - 2x = 2x - 2x - 1$; $-3x^2 - 2x = 0 - 1$; $-3x^2 - 2x = -1$; $-3x^2 - 2x + 1 = -1 + 1$; $-3x^2 - 2x + 1 = 0$

   Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

   Thus, $\boldsymbol{a = -3}$, $\boldsymbol{b = -2}$, and $\boldsymbol{c = 1}$

9. First - Write the quadratic equation $p^2 = p - 1$ in standard form $ap^2 + bp + c = 0$.

   $p^2 = p - 1$; $p^2 - p = p - p - 1$; $p^2 - p = 0 - 1$; $p^2 - p = -1$; $p^2 - p + 1 = -1 + 1$; $p^2 - p + 1 = 0$

Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

Thus, $\boldsymbol{a=1}$, $\boldsymbol{b=-1}$, and $\boldsymbol{c=1}$

10. First - Write the quadratic equation $3x-2=x^2$ in standard form $ax^2+bx+c=0$.

$3x-2=x^2$; $-x^2+3x-2=x^2-x^2$; $-x^2+3x-2=0$

Second - Equate the a, b, and c coefficients with the coefficients of the given quadratic equation.

Thus, $\boldsymbol{a=-1}$, $\boldsymbol{b=3}$, and $\boldsymbol{c=-2}$

Section 1.2 Case I Solutions - Solving Quadratic Equations of the Form ax^2+bx+c where $a=1$

1. $x^2=-5x-6$ Write the equation in standard form, i.e., $x^2+5x+6=0$.

Let: $a=1$, $b=5$, and $c=6$. Then,

Given: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\dfrac{-5\pm\sqrt{5^2-4\times1\times6}}{2\times1}$; $x=\dfrac{-5\pm\sqrt{25-24}}{2}$; $x=\dfrac{-5\pm\sqrt{1}}{2}$; $x=\dfrac{-5\pm1}{2}$ therefore,

I. $x=\dfrac{-5+1}{2}$; $x=-\dfrac{\overset{2}{\cancel{4}}}{\cancel{2}}$; $x=-\dfrac{2}{1}$; $\boldsymbol{x=-2}$ and

II. $x=\dfrac{-5-1}{2}$; $x=-\dfrac{\overset{3}{\cancel{6}}}{\cancel{2}}$; $x=-\dfrac{3}{1}$; $\boldsymbol{x=-3}$

Check: I. *Let* $x=-2$ *in* $x^2=-5x-6$; $(-2)^2\overset{?}{=}(-5\times-2)-6$; $4\overset{?}{=}10-6$; $4=4$

II. *Let* $x=-3$ *in* $x^2=-5x-6$; $(-3)^2\overset{?}{=}(-5\times-3)-6$; $9\overset{?}{=}15-6$; $9=9$

Therefore, the equation $x^2+5x+6=0$ can be factored to $\boldsymbol{(x+2)(x+3)=0}$.

2. $y^2-40y=-300$ Write the equation in standard form, i.e., $y^2-40y+300=0$.

Let: $a=1$, $b=-40$, and $c=300$. Then,

Given: $y=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $y=\dfrac{-(-40)\pm\sqrt{(-40)^2-4\times1\times300}}{2\times1}$; $y=\dfrac{40\pm\sqrt{1600-1200}}{2}$; $y=\dfrac{40\pm\sqrt{400}}{2}$

; $y=\dfrac{40\pm\sqrt{20^2}}{2}$; $y=\dfrac{40\pm20}{2}$ therefore, I. $y=\dfrac{40+20}{2}$; $y=\dfrac{\overset{30}{\cancel{60}}}{\cancel{2}}$; $y=\dfrac{30}{1}$; $\boldsymbol{y=30}$ and

II. $y=\dfrac{40-20}{2}$; $y=\dfrac{\overset{10}{\cancel{20}}}{\cancel{2}}$; $y=\dfrac{10}{1}$; $\boldsymbol{y=10}$

Check: I. *Let* $y=30$ *in* $y^2-40y=-300$; $(30)^2-40\cdot30\overset{?}{=}-300$; $900-1200\overset{?}{=}-300$; $-300=-300$

II. *Let* $y=10$ *in* $y^2-40y=-300$; $(10)^2-40\cdot10\overset{?}{=}-300$; $100-400\overset{?}{=}-300$; $-300=-300$

Therefore, the equation $y^2-40y+300=0$ can be factored to $\boldsymbol{(y-30)(y-10)=0}$.

3. $-x=-x^2+20$ Write the equation in standard form, i.e., $x^2-x-20=0$.

Let: $a=1$, $b=-1$, and $c=-20$. Then,

Given: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4\times1\times-20}}{2\times1}$; $x=\dfrac{1\pm\sqrt{1+80}}{2}$; $x=\dfrac{1\pm\sqrt{81}}{2}$; $x=\dfrac{1\pm\sqrt{9^2}}{2}$

; $x=\frac{1\pm 9}{2}$ therefore, I. $x=\frac{1+9}{2}$; $x=\frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $x=\frac{5}{1}$; $\boldsymbol{x=5}$ and

II. $x=\frac{1-9}{2}$; $x=-\frac{\overset{4}{\cancel{8}}}{\cancel{2}}$; $x=-\frac{4}{1}$; $\boldsymbol{x=-4}$

Check: I. *Let* $x=5$ *in* $-x=-x^2+20$; $-5\overset{?}{=}-5^2+20$; $-5\overset{?}{=}-25+20$; $-5=-5$

II. *Let* $x=-4$ *in* $-x=-x^2+20$; $-(-4)\overset{?}{=}-(-4)^2+20$; $4\overset{?}{=}-16+20$; $4=4$

Therefore, the equation $x^2-x-20=0$ can be factored to $\boldsymbol{(x-5)(x+4)=0}$.

4. $x^2+3x+4=0$ The equation is already in standard form.

Let: $a=1$, $b=3$, and $c=4$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-3\pm\sqrt{3^2-4\times 1\times 4}}{2\times 1}$; $x=\frac{-3\pm\sqrt{9-16}}{2}$; $x=\frac{-3\pm\sqrt{-7}}{2}$

Since the number under the radical is negative (an imaginary number), the given equation is not factorable.

5. $x^2-80-2x=0$ Write the equation in standard form, i.e., $x^2-2x-80=0$.

Let: $a=1$, $b=-2$, and $c=-80$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-(-2)\pm\sqrt{(-2)^2-4\times 1\times -80}}{2\times 1}$; $x=\frac{2\pm\sqrt{4+320}}{2}$; $x=\frac{2\pm\sqrt{324}}{2}$; $x=\frac{2\pm\sqrt{18^2}}{2}$

; $x=\frac{2\pm 18}{2}$ therefore, I. $x=\frac{2+18}{2}$; $x=\frac{\overset{10}{\cancel{20}}}{\cancel{2}}$; $x=\frac{10}{1}$; $\boldsymbol{x=10}$ and

II. $x=\frac{2-18}{2}$; $x=-\frac{\overset{8}{\cancel{16}}}{\cancel{2}}$; $x=-\frac{8}{1}$; $\boldsymbol{x=-8}$

Check: I. *Let* $x=10$ *in* $x^2-80-2x=0$; $10^2-80-2\cdot 10\overset{?}{=}0$; $100-80-20\overset{?}{=}0$; $100-100\overset{?}{=}0$; $0=0$

II. *Let* $x=-8$ *in* $x^2-80-2x=0$; $(-8)^2-80-2\cdot(-8)\overset{?}{=}0$; $64-80+16\overset{?}{=}0$; $80-80\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2-2x-80=0$ can be factored to $\boldsymbol{(x-10)(x+8)=0}$.

6. $x^2+4x+4=0$ The equation is already in standard form.

Let: $a=1$, $b=4$, and $c=4$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-4\pm\sqrt{4^2-4\times 1\times 4}}{2\times 1}$; $x=\frac{-4\pm\sqrt{16-16}}{2}$; $x=\frac{-4\pm\sqrt{0}}{2}$; $x=\frac{-4\pm 0}{2}$

; $x=-\frac{\overset{2}{\cancel{4}}}{\cancel{2}}$; $x=-\frac{2}{1}$; $x=-2$. In this case the equation has one repeated solution, i.e., $\boldsymbol{x=-2}$ and $\boldsymbol{x=-2}$.

Check: *Let* $x=-2$ *in* $x^2+4x+4=0$; $(-2)^2+4\cdot(-2)+4\overset{?}{=}0$; $4-8+4\overset{?}{=}0$; $8-8\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2+4x+4=0$ can be factored to $\boldsymbol{(x+2)(x+2)=0}$.

7. $-6=-w^2+w$ Write the equation in standard form, i.e., $w^2-w-6=0$

Let: $a=1$, $b=-1$, and $c=-6$. Then,

Given: $w=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $w=\dfrac{-(-1)\pm\sqrt{(-1)^2-4\times1\times-6}}{2\times1}$; $w=\dfrac{1\pm\sqrt{1+24}}{2}$; $w=\dfrac{1\pm\sqrt{25}}{2}$; $w=\dfrac{1\pm\sqrt{5^2}}{2}$

; $w=\dfrac{1\pm5}{2}$ therefore, I. $w=\dfrac{1+5}{2}$; $w=\dfrac{\overset{3}{\cancel{6}}}{\cancel{2}}$; $w=\dfrac{3}{1}$; $\boldsymbol{w=3}$ and

II. $w=\dfrac{1-5}{2}$; $w=-\dfrac{\overset{2}{\cancel{4}}}{\cancel{2}}$; $w=-\dfrac{2}{1}$; $\boldsymbol{w=-2}$

Check: I. *Let* $w=3$ *in* $-6=-w^2+w$; $-6\overset{?}{=}-\left(3^2\right)+3$; $-6\overset{?}{=}-9+3$; $-6=-6$

II. *Let* $w=-2$ *in* $-6=-w^2+w$; $-6\overset{?}{=}-(-2)^2+(-2)$; $-6\overset{?}{=}-4-2$; $-6=-6$

Therefore, the equation $w^2-w-6=0$ can be factored to $\boldsymbol{(w-3)(w+2)=0}$.

8. $4x=x^2$ Write the equation in standard form, i.e., $x^2-4x=0$.

Let: $a=1$, $b=-4$, and $c=0$. Then,

Given: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4\times1\times0}}{2\times1}$; $x=\dfrac{4\pm\sqrt{16-0}}{2}$; $x=\dfrac{4\pm\sqrt{16}}{2}$; $x=\dfrac{4\pm\sqrt{4^2}}{2}$

; $x=\dfrac{4\pm4}{2}$; therefore, I. $x=\dfrac{4-4}{2}$; $x=\dfrac{0}{2}$; $x=0$ and

II. $x=\dfrac{4+4}{2}$; $x=\dfrac{\overset{4}{\cancel{8}}}{\cancel{2}}$; $x=\dfrac{4}{1}$; $x=4$

Check: I. *Let* $x=0$ *in* $4x=x^2$; $4\cdot0=0^2$; $0=0$

II. *Let* $x=4$ *in* $4x=x^2$; $4\cdot4=4^2$; $16=16$

Therefore, the equation $x^2-4x=0$ can be factored to $(x+0)(x-4)=0$ which is the same as $x(x-4)=0$.

9. $z^2-37z-120=0$ The equation is already in standard form

Let: $a=1$, $b=-37$, and $c=-120$. Then,

Given: $z=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $z=\dfrac{-(-37)\pm\sqrt{(-37)^2-4\times1\times-120}}{2\times1}$; $z=\dfrac{37\pm\sqrt{1369+480}}{2}$; $z=\dfrac{37\pm\sqrt{1849}}{2}$

; $z=\dfrac{37\pm\sqrt{43^2}}{2}$; $z=\dfrac{37\pm43}{2}$ therefore, I. $z=\dfrac{37+43}{2}$; $z=\dfrac{\overset{40}{\cancel{80}}}{\cancel{2}}$; $z=\dfrac{40}{1}$; $\boldsymbol{z=40}$ and

II. $z=\dfrac{37-43}{2}$; $z=-\dfrac{\overset{3}{\cancel{6}}}{\cancel{2}}$; $z=-\dfrac{3}{1}$; $\boldsymbol{z=-3}$

Check: I. *Let* $z=40$ *in* $z^2-37z-120=0$; $40^2-37\cdot40-120\overset{?}{=}0$; $1600-1480-120\overset{?}{=}0$; $1600-1600\overset{?}{=}0$; $0=0$

II. *Let* $z=-3$ *in* $z^2-37z-120=0$; $(-3)^2-37\cdot(-3)-120\overset{?}{=}0$; $9+111-120\overset{?}{=}0$; $120-120\overset{?}{=}0$; $0=0$

Therefore, the equation $z^2-37z-120=0$ can be factored to $(z-\mathbf{40})(z+\mathbf{3})=\mathbf{0}$.

10. $x^2-20=-8x$ Write the equation in standard form, i.e., $x^2+8x-20=0$

Let: $a=1$, $b=8$, and $c=-20$. Then,

Given: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\dfrac{-8\pm\sqrt{8^2-4\times1\times-20}}{2\times1}$; $x=\dfrac{-8\pm\sqrt{64+80}}{2}$; $x=\dfrac{-8\pm\sqrt{144}}{2}$; $x=\dfrac{-8\pm\sqrt{12^2}}{2}$

; $x = \frac{-8 \pm 12}{2}$ therefore, I. $x = \frac{-8+12}{2}$; $x = \frac{\overset{2}{\cancel{4}}}{\cancel{2}}$; $x = \frac{2}{1}$; $\boldsymbol{x = 2}$ and

II. $x = \frac{-8-12}{2}$; $x = -\frac{\overset{10}{\cancel{20}}}{\cancel{2}}$; $x = -\frac{10}{1}$; $\boldsymbol{x = -10}$

Check: I. *Let* $x = 2$ *in* $x^2 - 20 = -8x$; $2^2 - 20 \overset{?}{=} -8 \cdot 2$; $4 - 20 \overset{?}{=} -16$; $-16 = -16$

II. *Let* $x = -10$ *in* $x^2 - 20 = -8x$; $(-10)^2 - 20 \overset{?}{=} -8 \cdot (-10)$; $100 - 20 \overset{?}{=} +80$; $80 = 80$

Therefore, the equation $x^2 + 8x - 20 = 0$ can be factored to $\boldsymbol{(x-2)(x+10) = 0}$.

Section 1.2 Case II Solutions - Solving Quadratic Equations of the Form $ax^2 + bx + c$ where $a \rangle 1$

1. $4u^2 + 6u + 1 = 0$ The quadratic equation is already in standard form.

Let: $a = 4$, $b = 6$, and $c = 1$. Then,

Given: $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $u = \frac{-6 \pm \sqrt{6^2 - 4 \times 4 \times 1}}{2 \times 4}$; $u = \frac{-6 \pm \sqrt{36-16}}{8}$; $u = \frac{-6 \pm \sqrt{20}}{8}$; $u = \frac{-6 \pm 4.47}{8}$

therefore, I. $u = \frac{-6 + 4.47}{8}$; $u = -\frac{1.53}{8}$; $\boldsymbol{u = -0.19}$ and

II. $u = \frac{-6 - 4.47}{8}$; $u = -\frac{10.47}{8}$; $\boldsymbol{u = -1.31}$

The solution set is $\{\boldsymbol{-1.31, -0.9}\}$.

Check: I. *Let* $u = -0.19$ *in* $4u^2 + 6u + 1 = 0$; $4 \cdot (-0.19)^2 + 6 \cdot -0.19 + 1 \overset{?}{=} 0$; $4 \cdot 0.036 - 1.14 + 1 \overset{?}{=} 0$; $0.14 - 1.14 + 1 \overset{?}{=} 0$

; $1.14 - 1.14 \overset{?}{=} 0$; $0 = 0$

II. *Let* $u = -1.31$ *in* $4u^2 + 6u + 1 = 0$; $4 \cdot (-1.31)^2 + 6 \cdot -1.31 + 1 \overset{?}{=} 0$; $4 \cdot 1.716 - 7.86 + 1 \overset{?}{=} 0$; $6.86 - 7.86 + 1 \overset{?}{=} 0$

; $7.86 - 7.86 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $4u^2 + 6u + 1 = 0$ can be factored to $\boldsymbol{(u + 0.19)(u + 1.31) = 0}$.

2. $4w^2 + 10w = -3$ Write the equation in standard form, i.e., $4w^2 + 10w + 3 = 0$.

Let: $a = 4$, $b = 10$, and $c = 3$. Then,

Given: $w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $w = \frac{-10 \pm \sqrt{10^2 - 4 \times 4 \times 3}}{2 \times 4}$; $w = \frac{-10 \pm \sqrt{100 - 48}}{8}$; $w = \frac{-10 \pm \sqrt{52}}{8}$

; $w = \frac{-10 \pm 7.2}{8}$ therefore, I. $w = \frac{-10 + 7.2}{8}$; $w = -\frac{2.8}{8}$; $\boldsymbol{w = -0.35}$ and

II. $w = \frac{-10 - 7.2}{8}$; $w = -\frac{17.2}{8}$; $\boldsymbol{w = -2.15}$

The solution set is $\{\boldsymbol{-2.15, -0.35}\}$.

Check: I. *Let* $w = -0.35$ *in* $4w^2 + 10w = -3$; $4 \cdot (-0.35)^2 + 10 \cdot -0.35 \overset{?}{=} -3$; $4 \cdot 0.123 - 3.5 \overset{?}{=} -3$; $0.5 - 3.5 \overset{?}{=} -3$

; $-3 = -3$

II. *Let* $w = -2.15$ *in* $4w^2 + 10w = -3$; $4 \cdot (-2.15)^2 + 10 \cdot -2.15 \overset{?}{=} -3$; $4 \cdot 4.62 - 21.5 \overset{?}{=} -3$; $18.5 - 21.5 \overset{?}{=} -3$

; $-3 = -3$

Therefore, the equation $4w^2 + 10w + 3 = 0$ can be factored to $\boldsymbol{(w + 0.35)(w + 2.15) = 0}$.

3. $6x^2 + 4x - 2 = 0$ The quadratic equation is already in standard form.

Let: $a=6$, $b=4$, and $c=-2$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-4\pm\sqrt{4^2-4\times6\times-2}}{2\times6}$; $x=\frac{-4\pm\sqrt{16+48}}{12}$; $x=\frac{-4\pm\sqrt{64}}{12}$; $x=\frac{-4\pm\sqrt{8^2}}{12}$

; $x=\frac{-4\pm8}{12}$ therefore, I. $x=\frac{-4+8}{12}$; $x=\frac{\cancel{4}}{\underset{3}{\cancel{12}}}$; $x=\frac{1}{3}$; $\boldsymbol{x=0.33}$ and

II. $x=\frac{-4-8}{12}$; $x=-\frac{\cancel{12}}{\cancel{12}}$; $x=-\frac{1}{1}$; $\boldsymbol{x=-1}$

The solution set is $\{\mathbf{-1,\ 0.33}\}$.

Check: I. *Let* $x=0.33$ *in* $6x^2+4x-2=0$; $6\cdot0.33^2+4\cdot0.33-2\overset{?}{=}0$; $6\cdot0.111+1.32-2\overset{?}{=}0$; $0.67+1.32-2\overset{?}{=}0$; $2-2\overset{?}{=}0$; $0=0$

II. *Let* $x=-1$ *in* $6x^2+4x-2=0$; $6\cdot(-1)^2+4\cdot-1-2\overset{?}{=}0$; $6\cdot1-4-2\overset{?}{=}0$; $6-6\overset{?}{=}0$; $0=0$

Therefore, the equation $6x^2+4x-2=0$ can be factored to $\boldsymbol{(x-0.33)(x+1)=0}$.

4. $15y^2+3=-14y$ Write the equation in standard form, i.e., $15y^2+14y+3=0$.

Let: $a=15$, $b=14$, and $c=3$. Then,

Given: $y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $y=\frac{-14\pm\sqrt{(-14)^2-4\times15\times3}}{2\times15}$; $y=\frac{-14\pm\sqrt{196-180}}{30}$; $y=\frac{-14\pm\sqrt{16}}{30}$; $y=\frac{-14\pm\sqrt{4^2}}{30}$

; $y=\frac{-14\pm4}{30}$ therefore, I. $y=\frac{-14+4}{30}$; $y=-\frac{\cancel{10}}{\underset{3}{\cancel{30}}}$; $y=-\frac{1}{3}$; $\boldsymbol{y=-0.33}$ and

II. $y=\frac{-14-4}{30}$; $y=-\frac{\overset{3}{\cancel{18}}}{\underset{5}{\cancel{30}}}$; $y=-\frac{3}{5}$; $\boldsymbol{y=-0.6}$

The solution set is $\{\mathbf{-0.6,\ -0.33}\}$.

Check I. *Let* $y=-0.33$ *in* $15y^2+3=-14y$; $15\cdot(-0.33)^2+3\overset{?}{=}-14\cdot-0.33$; $15\cdot0.108+3\overset{?}{=}4.62$; $1.62+3\overset{?}{=}4.62$; $4.62=4.62$

II. *Let* $y=-0.6$ *in* $15y^2+3=-14y$; $15\cdot(-0.6)^2+3\overset{?}{=}-14\cdot-0.6$; $15\cdot0.36+3\overset{?}{=}8.4$; $5.4+3\overset{?}{=}8.4$; $8.4=8.4$

Therefore, the equation $15y^2+3=-14y$ can be factored to $\boldsymbol{(y+0.6)(w+0.33)=0}$.

5. $2x^2-5x+3=0$ The equation is in standard form.

Let: $a=2$, $b=-5$, and $c=3$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-(-5)\pm\sqrt{(-5)^2-4\times2\times3}}{2\times2}$; $x=\frac{5\pm\sqrt{25-24}}{4}$; $x=\frac{5\pm\sqrt{1}}{4}$; $x=\frac{5\pm1}{4}$

therefore, I. $x=\frac{5+1}{4}$; $x=\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}$; $x=\frac{3}{2}$; $\boldsymbol{x=1.5}$ and

II. $x=\frac{5-1}{4}$; $x=\frac{\cancel{4}}{\cancel{4}}$; $x=\frac{1}{1}$; $\boldsymbol{x=1}$

The solution set is $\{\mathbf{1,\ 1.5}\}$.

Check I. *Let* $x=1$ *in* $2x^2-5x+3=0$; $2\cdot1^2-5\cdot1+3\overset{?}{=}0$; $2\cdot1-5+3\overset{?}{=}0$; $2-5+3\overset{?}{=}0$; $5-5\overset{?}{=}0$; $0=0$

II. *Let* $x=1.5$ *in* $2x^2-5x+3=0$; $2\cdot1.5^2-5\cdot1.5+3\overset{?}{=}0$; $2\cdot2.25-7.5+3\overset{?}{=}0$; $4.5-7.5+3\overset{?}{=}0$

$; \; 7.5 - 7.5 \overset{?}{=} 0 \; ; \; 0 = 0$

Therefore, the equation $2x^2 - 5x + 3 = 0$ can be factored to $\mathbf{(x-1)(x-1.5)=0}$.

6. $2x^2 + xy - y^2 = 0$ x is varaible Write the equation in standard form, i.e., $2x^2 + yx - y^2 = 0$.

Let: $a = 2$, $b = y$, and $c = -y^2$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-y \pm \sqrt{y^2 - 4 \times 2 \times -y^2}}{2 \times 2}$; $x = \frac{-y \pm \sqrt{y^2 + 8y^2}}{4}$; $x = \frac{-y \pm \sqrt{9y^2}}{4}$; $x = \frac{-y \pm 3y}{4}$

therefore, I. $x = \frac{-y + 3y}{4}$; $x = \frac{\cancel{2}y}{\underset{2}{\cancel{4}}}$; $x = \frac{1}{2}y$; $\boldsymbol{x = 0.5y}$ and

II. $x = \frac{-y - 3y}{4}$; $x = \frac{-4y}{4}$; $x = -\frac{\cancel{4}}{\cancel{4}}y$; $\boldsymbol{x = -y}$

The solution set is $\{\boldsymbol{-y, 0.5y}\}$.

I. *Let* $x = 0.5y$ *in* $2x^2 + xy - y^2 = 0$; $2 \cdot (0.5y)^2 + (0.5y) \cdot y - y^2 \overset{?}{=} 0$; $2 \cdot 0.25y^2 + 0.5y^2 - y^2 \overset{?}{=} 0$

$; \; 0.5y^2 + 0.5y^2 - y^2 \overset{?}{=} 0 \; ; \; y^2 - y^2 \overset{?}{=} 0 \; ; \; 0 = 0$

II. *Let* $x = -y$ *in* $2x^2 + xy - y^2 = 0$; $2 \cdot (-y)^2 + (-y) \cdot y - y^2 \overset{?}{=} 0$; $2y^2 - y^2 - y^2 \overset{?}{=} 0$; $2y^2 - 2y^2 \overset{?}{=} 0$; $0 = 0$

Therefore, the equation $2x^2 + xy - y^2 = 0$ can be factored to $\mathbf{(x+y)(x-0.5y)=0}$.

7. $6x^2 + 7x - 3 = 0$ The equation is already in standard form.

Let: $a = 6$, $b = 7$, and $c = -3$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-7 \pm \sqrt{7^2 - 4 \times 6 \times -3}}{2 \times 6}$; $x = \frac{-7 \pm \sqrt{49 + 72}}{12}$; $x = \frac{-7 \pm \sqrt{121}}{12}$; $x = \frac{-7 \pm \sqrt{11^2}}{12}$

; $x = \frac{-7 \pm 11}{12}$ therefore, I. $x = \frac{-7 + 11}{12}$; $x = \frac{\cancel{4}}{\underset{3}{\cancel{12}}}$; $x = \frac{1}{3}$; $\mathbf{x = 0.33}$ and

II. $x = \frac{-7 - 11}{12}$; $x = -\frac{\overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}}$; $x = -\frac{3}{2}$; $\mathbf{x = -1.5}$

The solution set is $\{\mathbf{-1.5, 0.33}\}$.

Check I. *Let* $x = 0.33$ *in* $6x^2 + 7x - 3 = 0$; $6 \cdot (0.33)^2 + 7 \cdot 0.33 - 3 \overset{?}{=} 0$; $6 \cdot 0.11 + 2.31 - 3 \overset{?}{=} 0$; $0.66 + 2.31 - 3 \overset{?}{=} 0$

$; \; 3 - 3 \overset{?}{=} 0 \; ; \; 0 = 0$

II. *Let* $x = -1.5$ *in* $6x^2 + 7x - 3 = 0$; $6 \cdot (-1.5)^2 + 7 \cdot -1.5 - 3 \overset{?}{=} 0$; $6 \cdot 2.25 - 10.5 - 3 \overset{?}{=} 0$; $13.5 - 10.5 - 3 \overset{?}{=} 0$

$; \; 13.5 - 13.5 \overset{?}{=} 0 \; ; \; 0 = 0$

Therefore, the equation $6x^2 + 7x - 3 = 0$ can be factored to $\mathbf{(x+1.5)(x-0.33)=0}$.

8. $5x^2 = -3x$ Write the equation in standard form, i.e., $5x^2 + 3x = 0$.

Let: $a = 5$, $b = 3$, and $c = 0$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 5 \times 0}}{2 \times 5}$; $x = \frac{-3 \pm \sqrt{9 - 0}}{10}$; $x = \frac{-3 \pm \sqrt{9}}{10}$; $x = \frac{-3 \pm \sqrt{3^2}}{10}$

; $x = \frac{-3 \pm 3}{10}$ therefore, I. $x = \frac{-3 + 3}{10}$; $x = \frac{0}{10}$; $\mathbf{x = 0}$ and

$$\text{II.}\quad x = \frac{-3-3}{10} \;;\; x = -\frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}} \;;\; \boldsymbol{x = -\frac{3}{5}}$$

Check: I. $\textit{Let } x = 0 \textit{ in } \quad 5x^2 = -3x \;;\; 5 \cdot 0^2 \overset{?}{=} -3 \cdot 0 \;;\; 5 \cdot 0 \overset{?}{=} -3 \cdot 0 \;;\; 0 = 0$

II. $\textit{Let } x = -\frac{3}{5} \textit{ in } \quad 5x^2 = -3x \;;\; 5 \cdot \left(-\frac{3}{5}\right)^2 \overset{?}{=} -3 \cdot -\frac{3}{5} \;;\; \cancel{5} \cdot \frac{9}{\underset{5}{\cancel{25}}} \overset{?}{=} +\frac{9}{5} \;;\; \frac{9}{5} = \frac{9}{5}$

The solution set is $\left\{ \boldsymbol{-\frac{3}{5}, 0} \right\}$.

Therefore, the equation $5x^2 + 3x = 0$ can be factored to $(x+0)\left(x + \frac{3}{5}\right) = 0$ which is the same as $\boldsymbol{x\left(x + \frac{3}{5}\right) = 0}$.

Note that this equation can further be simplified in order to obtain the original form of the quadratic equation as follows:

$; \; x\left(\frac{x}{1} + \frac{3}{5}\right) = 0 \;;\; x\left(\frac{(5 \cdot x) + (1 \cdot 3)}{1 \cdot 5}\right) = 0 \;;\; x\left(\frac{5x+3}{5}\right) = 0 \;;\; \frac{5x^2 + 3x}{5} = 0 \;;\; \frac{5x^2+3x}{5} = \frac{0}{1} \;;\; \left(5x^2 + 3x\right) \cdot 1 = 0 \cdot 5$

$; \; 5x^2 + 3x = 0$

9. $3x^2 + 4x + 5 = 0$ The equation is already in standard form

Let: $a = 3$, $b = 4$, and $c = 5$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \;;\; x = \frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times 5}}{2 \times 3} \;;\; x = \frac{-4 \pm \sqrt{16 - 60}}{6} \;;\; x = \frac{-4 \pm \sqrt{-44}}{6}$

Since the number under the radical is a negative number (an imaginary number) therefore, **the equation $\boldsymbol{3x^2 + 4x + 5 = 0}$ has no real solutions.**

10. $-3y^2 + 13y + 10 = 0$ The equation is in standard form.

Let: $a = -3$, $b = 13$, and $c = 10$. Then,

Given: $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \;;\; y = \frac{-13 \pm \sqrt{13^2 - 4 \times -3 \times 10}}{2 \times -3} \;;\; y = \frac{-13 \pm \sqrt{169 + 120}}{-6} \;;\; y = \frac{-13 \pm \sqrt{289}}{-6}$

$; \; y = \frac{-13 \pm \sqrt{17^2}}{-6} \;;\; y = \frac{-13 \pm 17}{-6}$ therefore, I. $y = \frac{-13+17}{-6} \;;\; y = \frac{\overset{2}{\cancel{4}}}{-\underset{3}{\cancel{6}}} \;;\; y = -\frac{2}{3} \;;\; \boldsymbol{y = -0.66}$ and

II. $y = \frac{-13-17}{-6} \;;\; y = \frac{-\overset{5}{\cancel{30}}}{-\cancel{6}} \;;\; y = \frac{5}{1} \;;\; \boldsymbol{y = 5}$

The solution set is $\{\boldsymbol{-0.66, 5}\}$.

Check I. $\textit{Let } y = 5 \textit{ in } \quad -3y^2 + 13y + 10 = 0 \;;\; -3 \cdot 5^2 + 13 \cdot 5 + 10 \overset{?}{=} 0 \;;\; -3 \cdot 25 + 65 + 10 \overset{?}{=} 0 \;;\; -75 + 65 + 10 \overset{?}{=} 0$

$; \; -75 + 75 \overset{?}{=} 0 \;;\; 0 = 0$

II. $\textit{Let } y = -0.66 \textit{ in } \quad -3y^2 + 13y + 10 = 0 \;;\; -3 \cdot (-0.66)^2 + 13 \cdot -0.66 + 10 \overset{?}{=} 0 \;;\; -3 \cdot 0.436 - 8.58 + 10 \overset{?}{=} 0$

$; \; -1.32 - 8.58 + 10 \overset{?}{=} 0 \;;\; -10 + 10 \overset{?}{=} 0 \;;\; 0 = 0$

Therefore, the equation $-3y^2 + 13y + 10 = 0$ can be factored to $\boldsymbol{(y + 0.66)(y - 5) = 0}$.

Section 1.3 Solutions - Solving Quadratic Equations Using the Square Root Property Method

1. First - Take the square root of both sides of the equation $(2y+5)^2 = 36$, i.e., $\sqrt{(2y+5)^2} = \pm\sqrt{36}$

Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{(2y+5)^2} = \pm\sqrt{36} \;;\; 2y + 5 = \pm 6$

Therefore the two solutions are: I. $2y+5=-6$; $2y=-6-5$; $2y=-11$; $\frac{2y}{2}=-\frac{11}{2}$; $y=-\frac{11}{2}$; $\mathbf{y=-5.5}$ and

II. $2y+5=+6$; $2y=6-5$; $2y=1$; $\frac{2y}{2}=\frac{1}{2}$; $y=\frac{1}{2}$; $\mathbf{y=0.5}$

Thus, the solution set is $\{\mathbf{-5.5, 0.5}\}$ and the equation $(2y+5)^2=36$ can be factored to $\mathbf{(y+5.5)(y-0.5)=0}$.

Check: I. *Let* $y=-5.5$ *in* $(2y+5)^2=36$; $(2\cdot -5.5+5)^2\stackrel{?}{=}36$; $(-11+5)^2\stackrel{?}{=}36$; $(-6)^2\stackrel{?}{=}36$; $6^2\stackrel{?}{=}36$; $36=36$

II. *Let* $y=0.5$ *in* $(2y+5)^2=36$; $(2\cdot 0.5+5)^2\stackrel{?}{=}36$; $(1+5)^2\stackrel{?}{=}36$; $6^2\stackrel{?}{=}36$; $36=36$

2. First - Take the square root of both sides of the equation $(x+1)^2=7$, i.e., $\sqrt{(x+1)^2}=\pm\sqrt{7}$

   Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{(x+1)^2}=\pm\sqrt{7}$; $x+1=\pm 2.65$

   Therefore the two solutions are: I. $x+1=-2.65$; $x=-2.65-1$; $\mathbf{x=-3.65}$ and

   II. $x+1=+2.65$; $x=2.65-1$; $\mathbf{x=1.65}$

   Thus, the solution set is $\{\mathbf{-3.65, 1.65}\}$ and the equation $(x+1)^2=7$ can be factored to $\mathbf{(x+3.65)(x-1.65)=0}$.

   Check: I. *Let* $x=-3.65$ *in* $(x+1)^2=7$; $(-3.65+1)^2\stackrel{?}{=}7$; $(-2.65)^2\stackrel{?}{=}7$; $7=7$

   II. *Let* $x=1.65$ *in* $(x+1)^2=7$; $(1.65+1)^2\stackrel{?}{=}7$; $2.65^2\stackrel{?}{=}7$; $7=7$

3. First - Take the square root of both sides of the equation $(2x-3)^2=1$, i.e., $\sqrt{(2x-3)^2}=\pm\sqrt{1}$

   Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{(2x-3)^2}=\pm\sqrt{1}$; $2x-3=\pm 1$

   Therefore the two solutions are: I. $2x-3=-1$; $2x=-1+3$; $2x=2$; $\frac{2x}{2}=\frac{2}{2}$; $x=\frac{1}{1}$; $\mathbf{x=1}$ and

   II. $2x-3=+1$; $2x=1+3$; $2x=4$; $\frac{2x}{2}=\frac{\overset{2}{4}}{2}$; $x=\frac{2}{1}$; $\mathbf{x=2}$

   Thus, the solution set is $\{\mathbf{1, 2}\}$ and the equation $(2x-3)^2=1$ can be factored to $\mathbf{(x-1)(x-2)=0}$.

   Check: I. *Let* $x=1$ *in* $(2x-3)^2=1$; $(2\cdot 1-3)^2\stackrel{?}{=}1$; $(2-3)^2\stackrel{?}{=}1$; $(-1)^2\stackrel{?}{=}1$; $1=1$

   II. *Let* $x=2$ *in* $(2x-3)^2=1$; $(2\cdot 2-3)^2\stackrel{?}{=}1$; $(4-3)^2\stackrel{?}{=}1$; $1^2\stackrel{?}{=}1$; $1=1$

4. First - Write the equation $x^2+3=0$ in the form of $x^2=b$, i.e., $x^2=-3$

   Second - Take the square root of both sides of the equation, i.e., $\sqrt{x^2}=\pm\sqrt{-3}$

   Since the number under the radical is a negative number (an imaginary number) therefore, **the equation $\mathbf{x^2+3=0}$ has no real solutions.**

5. First - Take the square root of both sides of the equation $(y-5)^2=5$, i.e., $\sqrt{(y-5)^2}=\pm\sqrt{5}$

   Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{(y-5)^2}=\pm\sqrt{5}$; $y-5=\pm 2.24$

   Therefore the two solutions are: I. $y-5=-2.24$; $y=-2.24+5$; $\mathbf{y=2.76}$ and

   II. $y-5=+2.24$; $y=2.24+5$; $\mathbf{y=7.24}$

   Thus, the solution set is $\{\mathbf{-2.76, 7.24}\}$ and the equation $(y-5)^2=5$ can be factored to $\mathbf{(y-2.76)(y-7.24)=0}$.

   Check: I. *Let* $y=2.76$ *in* $(y-5)^2=5$; $(2.76-5)^2\stackrel{?}{=}5$; $(-2.24)^2\stackrel{?}{=}5$; $5=5$

   II. *Let* $y=7.24$ *in* $(y-5)^2=5$; $(7.24-5)^2\stackrel{?}{=}5$; $(2.24)^2\stackrel{?}{=}5$; $5=5$

6. First - Write the equation $16x^2-25=0$ in the form of $ax^2=b$, i.e., $16x^2=25$

Second - Divide both sides of the equation $16x^2 = 25$ by the coefficient of x, i.e., $\frac{\cancel{16}x^2}{\cancel{16}} = \frac{25}{16}$; $x^2 = \frac{25}{16}$

Third - Take the square root of both sides of the equation, i.e., $\sqrt{x^2} = \pm\sqrt{\frac{25}{16}}$

Fourth - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{x = \pm\frac{5}{4}}$

Therefore, the solution set is $\left\{-\frac{\mathbf{5}}{\mathbf{4}}, \frac{\mathbf{5}}{\mathbf{4}}\right\}$ and the equation $16x^2 - 25 = 0$ can be factored to $\left(x - \frac{5}{4}\right)\left(x + \frac{5}{4}\right) = 0$ which is the same as $\boldsymbol{(4x - 5)(4x + 5) = 0}$.

Check: I. *Let* $x = -\frac{5}{4}$ *in* $16x^2 - 25 = 0$; $16 \cdot \left(-\frac{5}{4}\right)^2 - 25 \overset{?}{=} 0$; $\cancel{16} \cdot \frac{25}{\cancel{16}} - 25 \overset{?}{=} 0$; $25 - 25 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = -\frac{5}{4}$ *in* $16x^2 - 25 = 0$; $16 \cdot \left(\frac{5}{4}\right)^2 - 25 \overset{?}{=} 0$; $\cancel{16} \cdot \frac{25}{\cancel{16}} - 25 \overset{?}{=} 0$; $25 - 25 \overset{?}{=} 0$; $0 = 0$

7. First - Write the equation $x^2 - 49 = 0$ in the form of $x^2 = b$, i.e., $x^2 = 49$

Second - Take the square root of both sides of the equation, i.e., $\sqrt{x^2} = \pm\sqrt{49}$

Third - Simplify the terms on both sides to obtain the solutions, i.e., $\boldsymbol{x = \pm 7}$

Therefore, the solution set is $\{\mathbf{-7, 7}\}$ and the equation $x^2 = 49$ can be factored to $\boldsymbol{(x - 7)(x + 7) = 0}$.

Check: I. *Let* $x = -7$ *in* $x^2 - 49 = 0$; $(-7)^2 - 49 \overset{?}{=} 0$; $49 - 49 \overset{?}{=} 0$; $0 = 0$

II. *Let* $x = 7$ *in* $x^2 - 49 = 0$; $7^2 - 49 \overset{?}{=} 0$; $49 - 49 \overset{?}{=} 0$; $0 = 0$

8. First - Take the square root of both sides of the equation $(3x - 1)^2 = 25$, i.e., $\sqrt{(3x-1)^2} = \pm\sqrt{25}$

Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{(3x-1)^2} = \pm\sqrt{25}$; $3x - 1 = \pm 5$

Therefore, the two solutions are: I. $3x - 1 = -5$; $3x = -5 + 1$; $3x = -4$; $\frac{\cancel{3}x}{\cancel{3}} = -\frac{4}{3}$; $\boldsymbol{x = -1.33}$ and

II. $3x - 1 = +5$; $3x = 5 + 1$; $3x = 6$; $\frac{\cancel{3}x}{\cancel{3}} = \frac{\overset{2}{\cancel{6}}}{\cancel{3}}$; $x = \frac{2}{1}$; $\boldsymbol{x = 2}$

Thus, the solution set is $\{\mathbf{-1.33, 2}\}$ and the equation $(3x - 1)^2 = 25$ can be factored to $\boldsymbol{(x + 1.33)(x - 2) = 0}$.

Check: I. *Let* $x = -1.33$ *in* $(3x - 1)^2 = 25$; $(3 \cdot -1.33 - 1)^2 \overset{?}{=} 25$; $(-4 - 1)^2 \overset{?}{=} 25$; $(-5)^2 \overset{?}{=} 25$; $5^2 \overset{?}{=} 25$; $25 = 25$

II. *Let* $x = 2$ *in* $(3x - 1)^2 = 25$; $(3 \cdot 2 - 1)^2 \overset{?}{=} 25$; $(6 - 1)^2 \overset{?}{=} 25$; $5^2 \overset{?}{=} 25$; $25 = 25$

9. First - Take the square root of both sides of the equation $(x - 2)^2 = -7$, i.e., $\sqrt{(x-2)^2} = \pm\sqrt{-7}$

Since the number under the radical is a negative number (an imaginary number) therefore, **the equation $\boldsymbol{(x - 2)^2 = -7}$ has no real solutions.**

10. First - Take the square root of both sides of the equation $\left(x - \frac{1}{3}\right)^2 = \frac{1}{9}$, i.e., $\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{\frac{1}{9}}$

Second - Simplify the terms on both sides to obtain the solutions, i.e., $\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{\frac{1}{9}}$; $x - \frac{1}{3} = \pm\frac{1}{3}$

Therefore the two solutions are: I. $x - \frac{1}{3} = -\frac{1}{3}$; $x = -\frac{1}{3} + \frac{1}{3}$; $x = \frac{-1 + 1}{3}$; $x = \frac{0}{3}$; $\boldsymbol{x = 0}$ and

II. $x - \frac{1}{3} = +\frac{1}{3}$; $x = \frac{1}{3} + \frac{1}{3}$; $x = \frac{1 + 1}{3}$; $\boldsymbol{x = \frac{2}{3}}$

Thus, the solution set is $\left\{\mathbf{0}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$ and the equation $\left(x-\frac{1}{3}\right)^2=\frac{1}{9}$ can be factored to $(x+0)\left(x-\frac{2}{3}\right)=0$ which is the same as $x\left(x-\frac{2}{3}\right)=0$ or $\boldsymbol{x(3x-2)=0}$.

Check: I. *Let* $x=0$ *in* $\left(x-\frac{1}{3}\right)^2=\frac{1}{9}$; $\left(0-\frac{1}{3}\right)^2\overset{?}{=}\frac{1}{9}$; $\left(-\frac{1}{3}\right)^2\overset{?}{=}\frac{1}{9}$; $\frac{1}{9}=\frac{1}{9}$

II. *Let* $x=\frac{2}{3}$ *in* $\left(x-\frac{1}{3}\right)^2=\frac{1}{9}$; $\left(\frac{2}{3}-\frac{1}{3}\right)^2\overset{?}{=}\frac{1}{9}$; $\left(\frac{2-1}{3}\right)^2\overset{?}{=}\frac{1}{9}$; $\left(\frac{1}{3}\right)^2\overset{?}{=}\frac{1}{9}$; $\frac{1}{9}=\frac{1}{9}$

Section 1.4 Case I Solutions - Solving Quadratic Equations of the Form $ax^2+bx+c=0$, where $a=1$, by Completing the Square

1. First - Write the equation $x^2+10x-2=0$ in the form of $x^2+bx=-c$, i.e., $x^2+10x=2$.

Second - Complete the square and simplify. $x^2+10x=2$; $x^2+10x+\left(\frac{\cancelto{5}{10}}{2}\right)^2=2+\left(\frac{\cancelto{5}{10}}{2}\right)^2$; $x^2+10x+5^2=2+5^2$

; $x^2+10x+25=2+25$; $x^2+10x+25=27$; $(x+5)^2=27$

Third - Take the square root of both sides of the equation and solve for x.

$(x+5)^2=27$; $\sqrt{(x+5)^2}=\pm\sqrt{27}$; $x+5=\pm 5.19$. Therefore,

I. $x+5=+5.19$; $x=5.19-5$; $\boldsymbol{x=0.19}$ and II. $x+5=-5.19$; $x=-5.19-5$; $\boldsymbol{x=-10.19}$

The solution set is $\{\mathbf{-10.19, 0.19}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=0.19$ *in* $x^2+10x-2=0$; $0.19^2+10\cdot 0.19-2\overset{?}{=}0$; $0.036+1.9-2\overset{?}{=}0$; $2-2\overset{?}{=}0$; $0=0$

II. *Let* $x=-10.19$ *in* $x^2+10x-2=0$; $(-10.19)^2+10\cdot -10.19-2\overset{?}{=}0$; $103.8-101.9-2\overset{?}{=}0$; $101.9-101.9\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2+10x-2=0$ can be factored to $\boldsymbol{(x+10.19)(x-0.19)=0}$.

2. First - Write the equation $x^2-x-1=0$ in the form of $x^2+bx=-c$, i.e., $x^2-x=1$.

Second - Complete the square and simplify. $x^2-x=1$; $x^2-x+\left(-\frac{1}{2}\right)^2=1+\left(-\frac{1}{2}\right)^2$; $x^2-x+\frac{1}{4}=1+\frac{1}{4}$

; $\left(x-\frac{1}{2}\right)^2=1.25$

Third - Take the square root of both sides of the equation and solve for x.

$\left(x-\frac{1}{2}\right)^2=1.25$; $\sqrt{\left(x-\frac{1}{2}\right)^2}=\pm\sqrt{1.25}$; $x-\frac{1}{2}=\pm 1.118$; $x-0.5=\pm 1.118$. Therefore,

I. $x-0.5=+1.118$; $x=1.118+0.5$; $\boldsymbol{x=1.618}$ and II. $x-0.5=-1.118$; $x=-1.118+0.5$; $\boldsymbol{x=-0.618}$

The solution set is $\{\mathbf{-0.618, 1.618}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=1.618$ *in* $x^2-x-1=0$; $1.618^2-1.618-1\overset{?}{=}0$; $2.618-1.618-1\overset{?}{=}0$; $2.618-2.618\overset{?}{=}0$; $0=0$

II. *Let* $x=-0.618$ *in* $x^2-x-1=0$; $(-0.618)^2-(-0.618)-1\overset{?}{=}0$; $0.381+0.618-1\overset{?}{=}0$; $-0.618+0.618\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2-x-1=0$ can be factored to $\boldsymbol{(x+0.618)(x-1.618)=0}$.

3. First - Write the equation $x(x+2)=80$ in the form of $x^2+bx=-c$, i.e., $x^2+2x=80$.

Second - Complete the square and simplify. $x^2+2x=80$; $x^2+2x+\left(\frac{2}{2}\right)^2=80+\left(\frac{2}{2}\right)^2$; $x^2+2x+1=80+1$

; $(x+1)^2=81$

Third - Take the square root of both sides of the equation and solve for x.

$(x+1)^2=81$; $\sqrt{(x+1)^2}=\pm\sqrt{81}$; $x+1=\pm 9$. Therefore,

I. $x+1=+9$; $x=9-1$; $\boldsymbol{x=8}$ and II. $x+1=-9$; $x=-9-1$; $\boldsymbol{x=-10}$

The solution set is $\{\mathbf{-10,8}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=8$ *in* $x(x+2)=80$; $8(8+2)\overset{?}{=}80$; $8\cdot 10\overset{?}{=}80$; $80=80$

II. *Let* $x=-10$ *in* $x(x+2)=80$; $-10(-10+2)\overset{?}{=}80$; $-10\cdot -8\overset{?}{=}80$; $80=80$

Therefore, the equation $x(x+2)=80$ can be factored to $\boldsymbol{(x+10)(x-8)=0}$.

4. First - Write the equation $y^2-10y+5=0$ in the form of $y^2+by=-c$, i.e., $y^2-10y=-5$.

Second - Complete the square and simplify. $y^2-10y=-5$; $y^2-10y+\left(-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}\right)^2=-5+\left(-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}\right)^2$; $y^2-10y+5^2=-5+5^2$

; $y^2-10y+25=-5+25$; $y^2-10y+25=20$; $(y-5)^2=20$

Third - Take the square root of both sides of the equation and solve for y.

$(y-5)^2=20$; $\sqrt{(y-5)^2}=\pm\sqrt{20}$; $y-5=\pm 4.47$. Therefore,

I. $y-5=+4.47$; $y=4.47+5$; $\boldsymbol{y=9.47}$ and II. $y-5=-4.47$; $y=-4.47+5$; $\boldsymbol{y=0.53}$

The solution set is $\{\mathbf{0.53, 9.47}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $y=0.53$ *in* $y^2-10y+5=0$; $0.53^2-10\cdot 0.53+5\overset{?}{=}0$; $0.3-5.3+5\overset{?}{=}0$; $5.3-5.3\overset{?}{=}0$; $0=0$

II. *Let* $y=9.47$ *in* $y^2-10y+5=0$; $9.47^2-10\cdot 9.47+5\overset{?}{=}0$; $89.7-94.7+5\overset{?}{=}0$; $94.7-94.7\overset{?}{=}0$; $0=0$

Therefore, the equation $y^2-10y+5=0$ can be factored to $\boldsymbol{(y-0.53)(y-9.47)=0}$.

5. First - Write the equation $x^2+4x-5=0$ in the form of $x^2+bx=-c$, i.e., $x^2+4x=5$.

Second - Complete the square and simplify. $x^2+4x=5$; $x^2+4x+\left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)^2=5+\left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)^2$; $x^2+4x+2^2=5+2^2$

; $x^2+4x+4=5+4$; $x^2+4x+4=9$; $(x+2)^2=9$

Third - Take the square root of both sides of the equation and solve for x.

$(x+2)^2=9$; $\sqrt{(x+2)^2}=\pm\sqrt{9}$; $x+2=\pm 3$. Therefore,

I. $x+2=+3$; $x=3-2$; $\boldsymbol{x=1}$ and II. $x+2=-3$; $x=-2-3$; $\boldsymbol{x=-5}$

The solution set is $\{\mathbf{-5,1}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=1$ *in* $x^2+4x-5=0$; $1^2+4\cdot 1-5\overset{?}{=}0$; $1+4-5\overset{?}{=}0$; $5-5\overset{?}{=}0$; $0=0$

II. *Let* $x=-5$ *in* $x^2+4x-5=0$; $(-5)^2+4\cdot(-5)-5\overset{?}{=}0$; $25-20-5\overset{?}{=}0$; $25-25\overset{?}{=}0$; $0=0$

Therefore, the equation $x^2+4x-5=0$ can be factored to $\boldsymbol{(x+5)(x-1)=0}$.

6. The equation $y^2+4y=14$ is already in the form of $y^2+by=-c$.

First - Complete the square and simplify. $y^2+4y=14$; $y^2+4y+\left(\frac{\frac{4}{2}}{2}\right)^2=14+\left(\frac{\frac{4}{2}}{2}\right)^2$; $y^2+4y+2^2=14+2^2$

; $y^2+4y+4=14+4$; $y^2+4y+4=18$; $(y+2)^2=18$

Second - Take the square root of both sides of the equation and solve for y.

$(y+2)^2=8$; $\sqrt{(y+2)^2}=\pm\sqrt{18}$; $y+2=\pm 4.24$. Therefore,

I. $y+2=+4.24$; $y=4.24-2$; $\boldsymbol{y=2.24}$ and II. $y+2=-4.24$; $y=-4.24-2$; $\boldsymbol{y=-6.24}$

The solution set is $\{\mathbf{-6.24,\ 2.24}\}$.

Third - Check the answers and write the quadratic equation in its factored form.

I. *Let* $y=2.24$ *in* $y^2+4y=14$; $2.24^2+4\cdot 2.24\overset{?}{=}14$; $5+9\overset{?}{=}14$; $14=14$

II. *Let* $y=-6.24$ *in* $y^2+4y=14$; $(-6.24)^2+4\cdot -6.24\overset{?}{=}14$; $39-25\overset{?}{=}14$; $14=14$

Therefore, the equation $y^2+4y=14$ can be factored to $\boldsymbol{(y+6.24)(y-2.24)=0}$.

7. First - Write the equation $w^2+\frac{1}{3}w-\frac{1}{2}=0$ in the form of $w^2+bw=-c$, i.e., $w^2+\frac{1}{3}w=\frac{1}{2}$.

Second - Complete the square and simplify. $w^2+\frac{1}{3}w=\frac{1}{2}$; $w^2+\frac{1}{3}w+\left(\frac{1}{6}\right)^2=\frac{1}{2}+\left(\frac{1}{6}\right)^2$; $w^2+\frac{1}{3}w+\frac{1}{36}=\frac{1}{2}+\frac{1}{36}$

; $w^2+\frac{1}{3}w+\frac{1}{36}=\frac{(1\cdot 36)+(1\cdot 2)}{2\cdot 36}$; $w^2+\frac{1}{3}w+\frac{1}{36}=\frac{36+2}{72}$; $w^2+\frac{1}{3}w+\frac{1}{36}=\frac{38}{72}$; $\left(w+\frac{1}{6}\right)^2=\frac{38}{72}$

Third - Take the square root of both sides of the equation and solve for w.

$\left(w+\frac{1}{6}\right)^2=\frac{38}{72}$; $\sqrt{\left(w+\frac{1}{6}\right)^2}=\pm\sqrt{\frac{38}{72}}$; $w+\frac{1}{6}=\pm\sqrt{0.527}$; $w+0.167=\pm 0.726$. Therefore,

I. $w+0.167=+0.726$; $w=0.726-0.167$; $\boldsymbol{w=0.56}$ and II. $w+0.167=-0.726$; $w=-0.726-0.167$; $\boldsymbol{w=-0.89}$ The solution set is $\{\mathbf{-0.89,\ 0.56}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $w=0.56$ *in* $w^2+\frac{1}{3}w-\frac{1}{2}=0$; $0.56^2+\frac{1}{3}\cdot 0.56-\frac{1}{2}\overset{?}{=}0$; $0.31+0.19-0.5\overset{?}{=}0$; $0.5-0.5\overset{?}{=}0$; $0=0$

II. *Let* $w=-0.89$ *in* $w^2+\frac{1}{3}w-\frac{1}{2}=0$; $(-0.89)^2+\frac{1}{3}\cdot(-0.89)-\frac{1}{2}\overset{?}{=}0$; $0.79-0.29-0.5\overset{?}{=}0$; $0.79-0.79\overset{?}{=}0$; $0=0$

Therefore, the equation $w^2+\frac{1}{3}w-\frac{1}{2}=0$ can be factored to $\boldsymbol{(w+0.89)(w-0.56)=0}$.

8. The equation $z^2+3z=-\frac{1}{4}$ is already in the form of $z^2+bz=-c$.

First - Complete the square and simplify. $z^2+3z=-\frac{1}{4}$; $z^2+3z+\left(\frac{3}{2}\right)^2=-\frac{1}{4}+\left(\frac{3}{2}\right)^2$; $z^2+3z+\frac{9}{4}=-\frac{1}{4}+\frac{9}{4}$

; $z^2+3z+\frac{9}{4}=\frac{-1+9}{4}$; $z^2+3z+\frac{9}{4}=\frac{8}{4}$; $\left(z+\frac{3}{2}\right)^2=2$

Second - Take the square root of both sides of the equation and solve for z.

$\left(z+\frac{3}{2}\right)^2=2$; $\sqrt{\left(z+\frac{3}{2}\right)^2}=\pm\sqrt{2}$; $z+\frac{3}{2}=\pm 1.414$. Therefore,

I. $z+\frac{3}{2}=+1.414$; $z=-\frac{3}{2}+1.414$; $z=-1.5+1.414$; $\mathbf{z=-0.086}$ and II. $z+\frac{3}{2}=-1.414$; $z=-\frac{3}{2}-1.414$;

; $z=-1.5-1.414$; $\mathbf{z=-2.914}$ The solution set is $\{\mathbf{-2.914,\,-0.086}\}$.

Third - Check the answers and write the quadratic equation in its factored form.

I. $Let\ z=-0.1\ \ in\quad z^2+3z=-\frac{1}{4}$; $(-0.086)^2+3\cdot(-0.086)\stackrel{?}{=}-0.25$; $0.008-0.258\stackrel{?}{=}-0.25$; $-0.25=-0.25$

II. $Let\ z=-2.914\ \ in\quad z^2+3z=-\frac{1}{4}$; $(-2.914)^2+3\cdot(-2.914)\stackrel{?}{=}-0.25$; $8.49-8.74\stackrel{?}{=}-0.25$; $-0.25=-0.25$

Therefore, the equation $z^2+3z=-\frac{1}{4}$ can be factored to $\mathbf{(z+2.914)(z+0.086)=0}$.

9. First - Write the equation $z^2+\frac{5}{3}z-\frac{1}{2}=0$ in the form of $z^2+bz=-c$, i.e., $z^2+\frac{5}{3}z=\frac{1}{2}$.

Second - Complete the square and simplify. $z^2+\frac{5}{3}z=\frac{1}{2}$; $z^2+\frac{5}{3}z+\left(\frac{5}{6}\right)^2=\frac{1}{2}+\left(\frac{5}{6}\right)^2$; $z^2+\frac{5}{3}z+\frac{25}{36}=\frac{1}{2}+\frac{25}{36}$

; $z^2+\frac{5}{3}z+\frac{25}{36}=\frac{(1\cdot 36)+(25\cdot 2)}{2\cdot 36}$; $z^2+\frac{5}{3}z+\frac{25}{36}=\frac{36+50}{72}$; $z^2+\frac{5}{3}z+\frac{25}{36}=\frac{86}{72}$; $\left(z+\frac{5}{6}\right)^2=\frac{86}{72}$

Third - Take the square root of both sides of the equation and solve for z .

$\left(z+\frac{5}{6}\right)^2=\frac{86}{72}$; $\sqrt{\left(z+\frac{5}{6}\right)^2}=\pm\sqrt{\frac{86}{72}}$; $z+\frac{5}{6}=\pm\sqrt{1.194}$; $z+0.83=\pm 1.09$. Therefore,

I. $z+0.83=+1.09$; $z=1.09-0.83$; $\mathbf{z=0.26}$ and II. $z+0.83=-1.09$; $z=-1.09-0.83$; $\mathbf{z=-1.92}$

The solution set is $\{\mathbf{-1.92,\,0.26}\}$.

Fourth - Check the answers and write the quadratic equation in its factored form.

I. $Let\ z=0.26\ \ in\quad z^2+\frac{5}{3}z-\frac{1}{2}=0$; $0.26^2+\frac{5}{3}\cdot 0.26-\frac{1}{2}\stackrel{?}{=}0$; $0.07+0.43-0.5\stackrel{?}{=}0$; $0.5-0.5\stackrel{?}{=}0$; $0=0$

II. $Let\ z=-1.92\ \ in\quad z^2+\frac{5}{3}z-\frac{1}{2}=0$; $(-1.92)^2+\frac{5}{3}\cdot(-1.92)-\frac{1}{2}\stackrel{?}{=}0$; $3.7-3.2-0.5\stackrel{?}{=}0$; $3.7-3.7\stackrel{?}{=}0$; $0=0$

Therefore, the equation $z^2+\frac{5}{3}z-\frac{1}{2}=0$ can be factored to $\mathbf{(z+1.92)(z-0.26)=0}$.

10. The equation $x^2-6x=-4$ is already in the form of $x^2+bx=-c$.

First - Complete the square and simplify. $x^2-6x=-4$; $x^2-6x+\left(-\frac{\cancel{6}^{\,3}}{\cancel{2}}\right)^2=-4+\left(-\frac{\cancel{6}^{\,3}}{\cancel{2}}\right)^2$; $x^2-6x+3^2=-4+3^2$

; $x^2-6x+9=-4+9$; $x^2-6x+9=5$; $(x-3)^2=5$

Second - Take the square root of both sides of the equation and solve for x .

$(x-3)^2=5$; $\sqrt{(x-3)^2}=\pm\sqrt{5}$; $x-3=\pm 2.24$. Therefore,

I. $x-3=+2.24$; $x=2.24+3$; $\mathbf{x=5.24}$ and II. $x-3=-2.24$; $x=-2.24+3$; $\mathbf{x=0.76}$

The solution set is $\{\mathbf{0.76,\,5.24}\}$.

Third - Check the answers and write the quadratic equation in its factored form.

I. $Let\ x=0.76\ \ in\quad x^2-6x=-4$; $0.76^2-6\cdot 0.76\stackrel{?}{=}-4$; $0.57-4.57\stackrel{?}{=}-4$; $-4=-4$

II. $Let\ x=5.24\ \ in\quad x^2-6x=-4$; $5.24^2-6\cdot 5.24\stackrel{?}{=}-4$; $27.45-31.45\stackrel{?}{=}-4$; $-4=-4$

Therefore, the equation $x^2-6x=-4$ can be factored to $\mathbf{(x-0.76)(x-5.24)=0}$.

Section 1.4 Case II Solutions - Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$, where $a \rangle 1$, by Completing the Square

1. First - Write the equation $4u^2 + 6u + 1 = 0$ in the form of $au^2 + bu = -c$, i.e., $4u^2 + 6u = -1$.

   Second - Divide both sides of the equation by the coefficient of u^2, i.e., $\frac{\cancel{4}}{\cancel{4}}u^2 + \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}}u = -\frac{1}{4}$; $u^2 + \frac{3}{2}u = -\frac{1}{4}$

   Third - Complete the square and simplify. $u^2 + \frac{3}{2}u = -\frac{1}{4}$; $u^2 + \frac{3}{2}u + \left(\frac{3}{4}\right)^2 = -\frac{1}{4} + \left(\frac{3}{4}\right)^2$; $u^2 + \frac{3}{2}u + \frac{9}{16} = -\frac{1}{4} + \frac{9}{16}$

   ; $\left(u + \frac{3}{4}\right)^2 = \frac{(-1\cdot 16) + (9\cdot 4)}{4\cdot 16}$; $\left(u + \frac{3}{4}\right)^2 = \frac{-16 + 36}{64}$; $\left(u + \frac{3}{4}\right)^2 = \frac{\overset{5}{\cancel{20}}}{\underset{16}{\cancel{64}}}$; $\left(u + \frac{3}{4}\right)^2 = \frac{5}{16}$

   Fourth - Take the square root of both sides of the equation and solve for u.

   $\left(u + \frac{3}{4}\right)^2 = \frac{5}{16}$; $\sqrt{\left(u + \frac{3}{4}\right)^2} = \pm\sqrt{\frac{5}{16}}$; $u + \frac{3}{4} = \pm\sqrt{0.313}$; $u + 0.75 = \pm 0.56$. Therefore,

   I. $u + 0.75 = +0.56$; $u = 0.56 - 0.75$; $\boldsymbol{u = -0.19}$ and II. $u + 0.75 = -0.56$; $u = -0.56 - 0.75$; $\boldsymbol{u = -1.31}$

   The solution set is $\{\mathbf{-1.31, -0.19}\}$.

   Fifth - Check the answers and write the quadratic equation in its factored form.

   I. $Let\ u = -0.19\ in$ $4u^2 + 6u + 1 = 0$; $4\cdot(-0.19)^2 + 6\cdot -0.19 + 1 \overset{?}{=} 0$; $4\cdot 0.036 - 1.14 + 1 \overset{?}{=} 0$; $0.14 - 1.14 + 1 \overset{?}{=} 0$

   ; $1.14 - 1.14 \overset{?}{=} 0$; $0 = 0$

   II. $Let\ u = -1.31\ in$ $4u^2 + 6u + 1 = 0$; $4\cdot(-1.31)^2 + 6\cdot -1.31 + 1 \overset{?}{=} 0$; $4\cdot 1.716 - 7.86 + 1 \overset{?}{=} 0$; $6.86 - 7.86 + 1 \overset{?}{=} 0$

   ; $7.86 - 7.86 \overset{?}{=} 0$; $0 = 0$

   Therefore, the equation $4u^2 + 6u + 1 = 0$ can be factored to $\boldsymbol{(u + 1.31)(u + 0.19) = 0}$.

2. The equation $4w^2 + 10w = -3$ is already in standard form of $aw^2 + bw = -c$.

   First - Divide both sides of the equation by the coefficient of w^2, i.e., $\frac{\cancel{4}}{\cancel{4}}w^2 + \frac{\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}}w = -\frac{3}{4}$; $w^2 + \frac{5}{2}w = -\frac{3}{4}$

   Second - Complete the square and simplify. $w^2 + \frac{5}{2}w = -\frac{3}{4}$; $w^2 + \frac{5}{2}w + \left(\frac{5}{4}\right)^2 = -\frac{3}{4} + \left(\frac{5}{4}\right)^2$; $w^2 + \frac{5}{2}w + \frac{25}{16} = -\frac{3}{4} + \frac{25}{16}$

   ; $\left(w + \frac{5}{4}\right)^2 = \frac{(-3\cdot 16) + (25\cdot 4)}{4\cdot 16}$; $\left(w + \frac{5}{4}\right)^2 = \frac{-48 + 100}{64}$; $\left(w + \frac{5}{4}\right)^2 = \frac{\overset{13}{\cancel{52}}}{\underset{16}{\cancel{64}}}$; $\left(w + \frac{5}{4}\right)^2 = \frac{13}{16}$

   Third - Take the square root of both sides of the equation and solve for w.

   $\left(w + \frac{5}{4}\right)^2 = \frac{13}{16}$; $\sqrt{\left(w + \frac{5}{4}\right)^2} = \pm\sqrt{\frac{13}{16}}$; $w + \frac{5}{4} = \pm\sqrt{0.813}$; $w + 1.25 = \pm 0.9$. Therefore,

   I. $w + 1.25 = +0.9$; $w = 0.9 - 1.25$; $\boldsymbol{w = -0.35}$ and II. $w + 1.25 = -0.9$; $w = -0.9 - 1.25$; $\boldsymbol{w = -2.15}$

   The solution set is $\{\mathbf{-2.15, -0.35}\}$.

   Fourth - Check the answers and write the quadratic equation in its factored form.

   I. $Let\ w = -0.35\ in$ $4w^2 + 10w = -3$; $4\cdot(-0.35)^2 + 10\cdot -0.35 \overset{?}{=} -3$; $4\cdot 0.123 - 3.5 \overset{?}{=} -3$; $0.5 - 3.5 \overset{?}{=} -3$; $-3 = -3$

   II. $Let\ w = -2.15\ in$ $4w^2 + 10w = -3$; $4\cdot(-2.15)^2 + 10\cdot -2.15 \overset{?}{=} -3$; $4\cdot 4.62 - 21.5 \overset{?}{=} -3$; $18.5 - 21.5 \overset{?}{=} -3$; $-3 = -3$

   Therefore, the equation $4w^2 + 10w = -3$ can be factored to $\boldsymbol{(w + 2.15)(w + 0.35) = 0}$.

3. First - Write the equation $6x^2 + 4x - 2 = 0$ in the form of $ax^2 + bx = -c$, i.e., $6x^2 + 4x = 2$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{6}}{\cancel{6}}x^2+\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}}x=\frac{\cancel{2}}{\underset{3}{\cancel{6}}}$; $x^2+\frac{2}{3}x=\frac{1}{3}$

Third - Complete the square and simplify. $x^2+\frac{2}{3}x=\frac{1}{3}$; $x^2+\frac{2}{3}x+\left(\frac{\cancel{2}}{\underset{3}{\cancel{6}}}\right)^2=\frac{1}{3}+\left(\frac{\cancel{2}}{\underset{3}{\cancel{6}}}\right)^2$; $x^2+\frac{2}{3}x+\left(\frac{1}{3}\right)^2=\frac{1}{3}+\left(\frac{1}{3}\right)^2$

; $x^2+\frac{2}{3}x+\frac{1}{9}=\frac{1}{3}+\frac{1}{9}$; $\left(x+\frac{1}{3}\right)^2=\frac{(1\cdot 9)+(1\cdot 3)}{3\cdot 9}$; $\left(x+\frac{1}{3}\right)^2=\frac{9+3}{27}$; $\left(x+\frac{1}{3}\right)^2=\frac{12}{27}$

Fourth - Take the square root of both sides of the equation and solve for x.

$\left(x+\frac{1}{3}\right)^2=\frac{12}{27}$; $\sqrt{\left(x+\frac{1}{3}\right)^2}=\pm\sqrt{\frac{12}{27}}$; $x+\frac{1}{3}=\pm\sqrt{0.44}$; $x+0.33=\pm 0.66$. Therefore,

I. $x+0.33=+0.66$; $x=0.66-0.33$; $\boldsymbol{x=0.33}$ and II. $x+0.33=-0.66$; $x=-0.66-0.33$; $\boldsymbol{x=-1}$

The solution set is $\{\mathbf{-1,\ 0.33}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=0.33$ *in* $6x^2+4x-2=0$; $6\cdot(0.33)^2+4\cdot 0.33-2\overset{?}{=}0$; $6\cdot 0.11+1.32-2\overset{?}{=}0$; $0.66+1.32-2\overset{?}{=}0$

; $2-2\overset{?}{=}0$; $0=0$

II. *Let* $x=-1$ *in* $6x^2+4x-2=0$; $6\cdot(-1)^2+4\cdot -1-2\overset{?}{=}0$; $6\cdot 1-4-2\overset{?}{=}0$; $6-4-2\overset{?}{=}0$; $6-6\overset{?}{=}0$; $0=0$

Therefore, the equation $6x^2+4x-2=0$ can be factored to $\boldsymbol{(x-0.33)(x+1)=0}$.

4. First - Write the equation $15y^2+3=-14y$ in the form of $ay^2+by=-c$, i.e., $15y^2+14y=-3$.

Second - Divide both sides of the equation by the coefficient of y^2, i.e., $\frac{\cancel{15}}{\cancel{15}}y^2+\frac{14}{15}y=-\frac{\cancel{3}}{\underset{5}{\cancel{15}}}$; $y^2+\frac{14}{15}y=-\frac{1}{5}$

Third - Complete the square. $y^2+\frac{14}{15}y=-\frac{1}{5}$; $y^2+\frac{14}{15}y+\left(\frac{14}{30}\right)^2=-\frac{1}{5}+\left(\frac{14}{30}\right)^2$; $y^2+\frac{14}{15}y+\frac{196}{900}=-\frac{1}{5}+\frac{196}{900}$

; $\left(y+\frac{\overset{7}{\cancel{14}}}{\underset{15}{\cancel{30}}}\right)^2=\frac{(-1\cdot 900)+(196\cdot 5)}{5\cdot 900}$; $\left(y+\frac{7}{15}\right)^2=\frac{-900+980}{4500}$; $\left(y+\frac{7}{15}\right)^2=\frac{\overset{4}{\cancel{80}}}{\underset{225}{\cancel{4500}}}$; $\left(y+\frac{7}{15}\right)^2=\frac{4}{225}$

Fourth - Take the square root of both sides of the equation and solve for y.

$\left(y+\frac{7}{15}\right)^2=\frac{4}{225}$; $\sqrt{\left(y+\frac{7}{15}\right)^2}=\pm\sqrt{\frac{4}{225}}$; $y+\frac{7}{15}=\pm\sqrt{0.02}$; $y+0.46=\pm 0.13$. Therefore,

I. $y+0.46=+0.13$; $y=0.13-0.46$; $\boldsymbol{y=-0.33}$ and II. $y+0.46=-0.13$; $y=-0.13-0.46$; $\boldsymbol{y=-0.59}$

The solution set is $\{\mathbf{-0.59,\ -0.33}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $y=-0.33$ *in* $15y^2+3=-14y$; $15\cdot(-0.33)^2+3\overset{?}{=}-14\cdot -0.33$; $15\cdot 0.108+3\overset{?}{=}4.62$; $1.62+3\overset{?}{=}4.62$

; $4.62=4.62$

II. *Let* $y=-0.59$ *in* $15y^2+3=-14y$; $15\cdot(-0.59)^2+3\overset{?}{=}-14\cdot -0.59$; $15\cdot 0.348+3\overset{?}{=}8.26$; $5.23+3\overset{?}{=}8.26$

; $8.26=8.26$

Therefore, the equation $15y^2+3=-14y$ can be factored to $\boldsymbol{(y+0.59)(w+0.33)=0}$.

5. First - Write the equation $2x^2-5x+3=0$ in the form of $ax^2+bx=-c$, i.e., $2x^2-5x=-3$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{2}}{\cancel{2}}x^2-\frac{5}{2}x=-\frac{3}{2}$; $x^2-\frac{5}{2}x=-\frac{3}{2}$

Third - Complete the square and simplify. $x^2-\frac{5}{2}x=-\frac{3}{2}$; $x^2-\frac{5}{2}x+\left(-\frac{5}{4}\right)^2=-\frac{3}{2}+\left(-\frac{5}{4}\right)^2$; $x^2-\frac{5}{2}x+\frac{25}{16}=-\frac{3}{2}+\frac{25}{16}$

; $\left(x-\frac{5}{4}\right)^2=\frac{(-3\cdot 16)+(2\cdot 25)}{2\cdot 16}$; $\left(x-\frac{5}{4}\right)^2=\frac{-48+50}{32}$; $\left(x-\frac{5}{4}\right)^2=\frac{\cancel{2}}{\cancel{32}_{16}}$; $\left(x-\frac{5}{4}\right)^2=\frac{1}{16}$

Fourth - Take the square root of both sides of the equation and solve for x.

$\left(x-\frac{5}{4}\right)^2=\frac{1}{16}$; $\sqrt{\left(x-\frac{5}{4}\right)^2}=\pm\sqrt{\frac{1}{16}}$; $x-\frac{5}{4}=\pm\frac{1}{4}$; $x-1.25=\pm 0.25$. Therefore,

I. $x-1.25=+0.25$; $x=0.25+1.25$; $\boldsymbol{x=1.5}$ and II. $x-1.25=-0.25$; $x=-0.25+1.25$; $\boldsymbol{x=1}$

The solution set is $\{\mathbf{1, 1.5}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=1$ *in* $2x^2-5x+3=0$; $2\cdot 1^2-5\cdot 1+3\overset{?}{=}0$; $2\cdot 1-5+3\overset{?}{=}0$; $2-5+3\overset{?}{=}0$; $5-5\overset{?}{=}0$; $0=0$

II. *Let* $x=1.5$ *in* $2x^2-5x+3=0$; $2\cdot 1.5^2-5\cdot 1.5+3\overset{?}{=}0$; $2\cdot 2.25-7.5+3\overset{?}{=}0$; $4.5-7.5+3\overset{?}{=}0$; $7.5-7.5\overset{?}{=}0$; $0=0$

Therefore, the equation $2x^2-5x+3=0$ can be factored to $\boldsymbol{(x-1)(x-1.5)=0}$.

6. First - Write the equation $2x^2+xy-y^2=0$, where x is variable, in the form of $ax^2+bx=-c$, i.e., $2x^2+yx=y^2$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{2}}{\cancel{2}}x^2+\frac{y}{2}x=\frac{y^2}{2}$; $x^2+\frac{y}{2}x=\frac{y^2}{2}$

Third - Complete the square and simplify. $x^2+\frac{y}{2}x=\frac{y^2}{2}$; $x^2+\frac{y}{2}x+\left(\frac{y}{4}\right)^2=\frac{y^2}{2}+\left(\frac{y}{4}\right)^2$; $x^2+\frac{y}{2}x+\left(\frac{y}{4}\right)^2=\frac{y^2}{2}+\frac{y^2}{16}$

; $\left(x+\frac{y}{4}\right)^2=\frac{(y^2\cdot 16)+(y^2\cdot 2)}{2\cdot 16}$; $\left(x+\frac{y}{4}\right)^2=\frac{16y^2+2y^2}{32}$; $\left(x+\frac{y}{4}\right)^2=\frac{\cancel{18}^{9}}{\cancel{32}_{16}}y^2$; $\left(x+\frac{y}{4}\right)^2=\frac{9}{16}y^2$

Fourth - Take the square root of both sides of the equation and solve for x.

$\left(x+\frac{y}{4}\right)^2=\frac{9}{16}y^2$; $\sqrt{\left(x+\frac{y}{4}\right)^2}=\pm\sqrt{\frac{9}{16}y^2}$; $x+\frac{y}{4}=\pm\frac{3}{4}y$; $x+0.25y=\pm 0.75y$. Therefore,

I. $x+0.25y=+0.75y$; $x=0.75y-0.25y$; $\boldsymbol{x=0.5y}$ and II. $x+0.25y=-0.75y$; $x=-0.75y-0.25y$; $\boldsymbol{x=-y}$

The solution set is $\{\boldsymbol{-y, 0.5y}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=0.5y$ *in* $2x^2+xy-y^2=0$; $2\cdot(0.5y)^2+(0.5y)\cdot y-y^2\overset{?}{=}0$; $2\cdot 0.25y^2+0.5y^2-y^2\overset{?}{=}0$

; $0.5y^2+0.5y^2-y^2\overset{?}{=}0$; $y^2-y^2\overset{?}{=}0$; $0=0$

II. *Let* $x=-y$ *in* $2x^2+xy-y^2=0$; $2\cdot(-y)^2+(-y)\cdot y-y^2\overset{?}{=}0$; $2y^2-y^2-y^2\overset{?}{=}0$; $2y^2-2y^2\overset{?}{=}0$; $0=0$

Therefore, the equation $2x^2+xy-y^2=0$ can be factored to $\boldsymbol{(x+y)(x-0.5y)=0}$.

7. First - Write the equation $6x^2+7x-3=0$ in the form of $ax^2+bx=-c$, i.e., $6x^2+7x=3$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{6}}{\cancel{6}}x^2+\frac{7}{6}x=\frac{\cancel{3}^{1}}{\cancel{6}_{2}}$; $x^2+\frac{7}{6}x=\frac{1}{2}$

Third - Complete the square and simplify. $x^2+\frac{7}{6}x=\frac{1}{2}$; $x^2+\frac{7}{6}x+\left(\frac{7}{12}\right)^2=\frac{1}{2}+\left(\frac{7}{12}\right)^2$; $x^2+\frac{7}{6}x+\frac{49}{144}=\frac{1}{2}+\frac{49}{144}$

; $\left(x+\frac{7}{12}\right)^2=\frac{(1\cdot 144)+(49\cdot 2)}{2\cdot 144}$; $\left(x+\frac{7}{12}\right)^2=\frac{144+98}{288}$; $\left(x+\frac{7}{12}\right)^2=\frac{242}{288}$

Fourth - Take the square root of both sides of the equation and solve for x.

$\left(x+\frac{7}{12}\right)^2=\frac{242}{288}$; $\sqrt{\left(x+\frac{7}{12}\right)^2}=\pm\sqrt{\frac{242}{288}}$; $x+\frac{7}{12}=\pm\sqrt{0.84}$; $x+0.58=\pm0.92$. Therefore,

I. $x+0.58=+0.92$; $x=0.92-0.58$; $\boldsymbol{x=0.34}$ and II. $x+0.58=-0.92$; $x=-0.92-0.58$; $\boldsymbol{x=-1.5}$

The solution set is $\{\mathbf{-1.5, 0.34}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=0.34$ *in* $6x^2+7x-3=0$; $6\cdot(0.34)^2+7\cdot0.34-3\overset{?}{=}0$; $6\cdot0.115+2.38-3\overset{?}{=}0$; $0.69+2.38-3\overset{?}{=}0$; $3-3\overset{?}{=}0$; $0=0$

II. *Let* $x=-1.5$ *in* $6x^2+7x-3=0$; $6\cdot(-1.5)^2+7\cdot-1.5-3\overset{?}{=}0$; $6\cdot2.25-10.5-3\overset{?}{=}0$; $13.5-10.5-3\overset{?}{=}0$; $13.5-13.5\overset{?}{=}0$; $0=0$

Therefore, the equation $6x^2+7x-3=0$ can be factored to $\boldsymbol{(x+1.5)(x-0.34)=0}$.

8. First - Write the equation $5x^2=-3x$ in the form of $ax^2+bx=-c$, i.e., $5x^2+3x=0$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{5}}{\cancel{5}}x^2+\frac{3}{5}x=\frac{0}{5}$; $x^2+\frac{3}{5}x=0$

Third - Complete the square and simplify. $x^2+\frac{3}{5}x=0$; $x^2+\frac{3}{5}x+\left(\frac{3}{10}\right)^2=0+\left(\frac{3}{10}\right)^2$; $x^2+\frac{3}{5}u+\frac{9}{100}=\frac{9}{100}$; $\left(x+\frac{3}{10}\right)^2=\frac{9}{100}$

Fourth - Take the square root of both sides of the equation and solve for x.

$\left(x+\frac{3}{10}\right)^2=\frac{9}{100}$; $\sqrt{\left(x+\frac{3}{10}\right)^2}=\pm\sqrt{\frac{9}{100}}$; $x+\frac{3}{10}=\pm\frac{3}{10}$. Therefore,

I. $x+\frac{3}{10}=+\frac{3}{10}$; $x=\frac{3}{10}-\frac{3}{10}$; $\boldsymbol{x=0}$ and

II. $x+\frac{3}{10}=-\frac{3}{10}$; $x=-\frac{3}{10}-\frac{3}{10}$; $x=\frac{-3-3}{10}$; $x=-\frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}}$; $x=-\frac{3}{5}$; $\boldsymbol{x=-\frac{3}{5}}$

The solution set is $\left\{\boldsymbol{-\frac{3}{5}, 0}\right\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $x=0$ *in* $5x^2=-3x$; $5\cdot0^2\overset{?}{=}-3\cdot0$; $5\cdot0\overset{?}{=}-3\cdot0$; $0=0$

II. *Let* $x=-\frac{3}{5}$ *in* $4x^2=-3x$; $5\cdot\left(-\frac{3}{5}\right)^2\overset{?}{=}-3\cdot-\frac{3}{5}$; $\cancel{5}\cdot\frac{9}{\underset{5}{\cancel{25}}}\overset{?}{=}\frac{9}{5}$; $\frac{9}{5}=\frac{9}{5}$

Therefore, the equation $5x^2=-3x$ can be factored to $(x+0)\left(x+\frac{3}{5}\right)=0$ which is the same as $\boldsymbol{x(5x+3)=0}$.

9. First - Write the equation $3x^2+4x+5=0$ in the form of $ax^2+bx=-c$, i.e., $3x^2+4x=-5$.

Second - Divide both sides of the equation by the coefficient of x^2, i.e., $\frac{\cancel{3}}{\cancel{3}}x^2+\frac{4}{3}x=-\frac{5}{3}$; $x^2+\frac{4}{3}x=-\frac{5}{3}$

Third - Complete the square and simplify. $x^2+\frac{4}{3}x=-\frac{5}{3}$; $x^2+\frac{4}{3}x+\left(\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}}\right)^2=-\frac{5}{3}+\left(\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}}\right)^2$; $x^2+\frac{4}{3}x+\left(\frac{2}{3}\right)^2=-\frac{5}{3}+\left(\frac{2}{3}\right)^2$

; $x^2+\frac{4}{3}x+\frac{4}{9}=-\frac{5}{3}+\frac{4}{9}$; $\left(x+\frac{2}{3}\right)^2=\frac{(-5\cdot9)+(4\cdot3)}{3\cdot9}$; $\left(x+\frac{2}{3}\right)^2=\frac{-45+12}{27}$; $\left(x+\frac{2}{3}\right)^2=-\frac{\overset{11}{\cancel{33}}}{\underset{9}{\cancel{27}}}$; $\left(x+\frac{2}{3}\right)^2=-\frac{11}{9}$

Fourth - Take the square root of both sides of the equation and solve for x.

$$\left(x+\frac{2}{3}\right)^2=-\frac{11}{9}\ ;\ \sqrt{\left(x+\frac{2}{3}\right)^2}=\pm\sqrt{-\frac{11}{9}}\ ;\ x+\frac{2}{3}=\pm\sqrt{-1.22}$$

Since the number under the radical is a negative number (an imaginary number) therefore, **the equation $3x^2+4x+5=0$ has no real solutions.**

10. First - Write the equation $-3y^2+13y+10=0$ in the form of $ay^2+by=-c$, i.e., $-3y^2+13y=-10$.

Second - Divide both sides of the equation by the coefficient of y^2, i.e., $\frac{\cancel{-3}}{\cancel{-3}}y^2+\frac{13}{-3}y=\frac{-10}{-3}$; $y^2-\frac{13}{3}y=\frac{10}{3}$

Third - Complete the square and simplify. $y^2-\frac{13}{3}y=\frac{10}{3}$; $y^2-\frac{13}{3}y+\left(-\frac{13}{6}\right)^2=\frac{10}{3}+\left(-\frac{13}{6}\right)^2$

; $y^2-\frac{13}{3}y+\frac{169}{36}=\frac{10}{3}+\frac{169}{36}$; $\left(y-\frac{13}{6}\right)^2=\frac{(10\cdot 36)+(169\cdot 3)}{3\cdot 36}$; $\left(y-\frac{13}{6}\right)^2=\frac{360+507}{108}$; $\left(y-\frac{13}{6}\right)^2=\frac{867}{108}$

Fourth - Take the square root of both sides of the equation and solve for y.

$\left(y-\frac{13}{6}\right)^2=\frac{867}{108}$; $\sqrt{\left(y-\frac{13}{6}\right)^2}=\pm\sqrt{\frac{867}{108}}$; $y-\frac{13}{6}=\pm\sqrt{8.03}$; $y-2.17=\pm 2.83$. Therefore,

I. $y-2.17=+2.83$; $y=2.83+2.17$; $\boldsymbol{y=5}$ and II. $y-2.17=-2.83$; $y=-2.83+2.17$; $\boldsymbol{y=-0.66}$

The solution set is $\{\mathbf{-0.66,\ 5}\}$.

Fifth - Check the answers and write the quadratic equation in its factored form.

I. *Let* $y=5$ *in* $-3y^2+13y+10=0$; $-3\cdot 5^2+13\cdot 5+10\stackrel{?}{=}0$; $-3\cdot 25+65+10\stackrel{?}{=}0$; $-75+65+10\stackrel{?}{=}0$

; $-75+75\stackrel{?}{=}0$; $0=0$

II. *Let* $y=-0.66$ *in* $-3y^2+13y+10=0$; $-3\cdot(-0.66)^2+13\cdot -0.66+10\stackrel{?}{=}0$; $-3\cdot 0.436-8.58+10\stackrel{?}{=}0$

; $-1.32-8.58+10\stackrel{?}{=}0$; $-10+10\stackrel{?}{=}0$; $0=0$

Therefore, the equation $-3y^2+13y+10=0$ can be factored to $\boldsymbol{(y+0.66)(y-5)=0}$.

Section 1.5 Case I Solutions - Solving Quadratic Equations Containing Radicals

1. First - Move $-y+2$ terms of the equation $\sqrt{-9y+28}-y+2=0$ to the right hand side of the equation to obtain $\sqrt{-9y+28}=y-2$

Second - Square both sides of the equation $\sqrt{-9y+28}=y-2$; $\left(\sqrt{-9y+28}\right)^2=(y-2)^2$; $-9y+28=(y-2)^2$

Third - Complete the square on the right hand side of the equation and simplify. $-9y+28=(y-2)^2$

; $-9y+28=y^2+4-4y$; $0=y^2+(4-28)+(9y-4y)$; $0=y^2-24+5y$

Fourth - Write the quadratic equation $0=y^2-24+5y$ in standard form, i.e., $y^2+5y-24=0$

Fifth - Solve the quadratic equation by choosing a solution method. $y^2+5y-24=0$; $(y-3)(y+8)=0$.

Therefore, the two apparent solutions are: $y-3=0$; $y=3$ and $y+8=0$; $y=-8$

Sixth - Check the answers by substituting the y values into the original equation.

I. *Let* $y=3$ *in* $\sqrt{-9y+28}-y+2=0$; $\sqrt{-9\cdot 3+28}-3+2\stackrel{?}{=}0$; $\sqrt{-27+28}-1\stackrel{?}{=}0$; $\sqrt{1}-1\stackrel{?}{=}0$; $1-1\stackrel{?}{=}0$; $0=0$

II. *Let* $y=-8$ *in* $\sqrt{-9y+28}-y+2=0$; $\sqrt{-9\cdot(-8)+28}-(-8)+2\stackrel{?}{=}0$; $\sqrt{72+28}+8+2\stackrel{?}{=}0$; $\sqrt{100}+10\stackrel{?}{=}0$

; $\sqrt{10^2}+10\stackrel{?}{=}0$; $10+10\stackrel{?}{=}0$; $20\neq 0$

Therefore, $\boldsymbol{y=3}$ is the only real solution to the equation $\sqrt{-9y+28}-y+2=0$.

2. First - Square both sides of the equation. $2x=\sqrt{9x+3}$; $(2x)^2=\left(\sqrt{9x+3}\right)^2$; $4x^2=9x+3$

Second - Write the quadratic equation $4x^2=9x+3$ in standard form, i.e., $4x^2-9x-3=0$

Third - Solve the quadratic equation by choosing a solution method. $4x^2-9x-3=0$; $(x+0.3)(x-2.55)=0$.

Therefore, the two apparent solutions are: $x+0.3=0$; $x=-0.3$ and $x-2.55=0$; $x=2.55$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=-0.3$ *in* $2x=\sqrt{9x+3}$; $2\cdot(-0.3)\stackrel{?}{=}\sqrt{9\cdot(-0.3)+3}$; $-0.6\stackrel{?}{=}\sqrt{-2.7+3}$; $-0.6\stackrel{?}{=}\sqrt{0.3}$; $-0.6\neq 0.547$

II. *Let* $x=2.55$ *in* $2x=\sqrt{9x+3}$; $2\cdot 2.55\stackrel{?}{=}\sqrt{9\cdot 2.55+3}$; $5.1\stackrel{?}{=}\sqrt{22.95+3}$; $5.1\stackrel{?}{=}\sqrt{25.95}$; $5.1=5.1$

Therefore, the equation $2x=\sqrt{9x+3}$ has one real solution, i.e., $\boldsymbol{x=2.55}$.

3. First - Write the quadratic equation $t^2=-\sqrt{5}t$ in standard form, i.e., $t^2+\sqrt{5}t=0$

Second - Solve the quadratic equation by choosing a solution method. $t^2+\sqrt{5}t=0$; $t\left(t+\sqrt{5}\right)=0$.

Therefore, the two apparent solutions are: $t=0$ and $t+\sqrt{5}=0$; $t=-\sqrt{5}$

Third - Check the answers by substituting the t values into the original equation.

I. *Let* $t=0$ *in* $t^2=-\sqrt{5}t$; $0^2\stackrel{?}{=}-\sqrt{5}\cdot 0$; $0=0$

II. *Let* $t=0$ *in* $t^2=-\sqrt{5}t$; $\left(-\sqrt{5}\right)^2\stackrel{?}{=}-\sqrt{5}\cdot-\sqrt{5}$; $+5\stackrel{?}{=}+\sqrt{5\cdot 5}$; $5\stackrel{?}{=}\sqrt{5^2}$; $5=5$

Therefore, $\boldsymbol{t=0}$ and $\boldsymbol{t=-\sqrt{5}}$ are the real solutions to $t^2=-\sqrt{5}t$. Furthermore, the equation $t^2=-\sqrt{5}t$ can be factored to $\boldsymbol{t\left(t+\sqrt{5}\right)=0}$.

4. First - Write the quadratic equation $y^2-\sqrt{8}y=7$ in standard form. $y^2-\sqrt{8}y-7=0$

Second - Solve the quadratic equation by choosing a solution method. $y^2-\sqrt{8}y-7=0$; $(y+1.6)(y-4.4)=0$.

Therefore, the two apparent solutions are: $y+1.6=0$; $y=-1.6$ and $y-4.4=0$; $y=4.4$

Third - Check the answers by substituting the y values into the original equation.

I. *Let* $y=-1.6$ *in* $y^2-\sqrt{8}y=7$; $(-1.6)^2-\sqrt{8}\cdot(-1.6)\stackrel{?}{=}7$; $2.56-2.83\cdot(-1.6)\stackrel{?}{=}7$; $2.56+4.53\stackrel{?}{=}7$; $7=7$

II. *Let* $y=4.4$ *in* $y^2-\sqrt{8}y=7$; $4.4^2-\sqrt{8}\cdot 4.4\stackrel{?}{=}7$; $19.36-2.83\cdot 4.4\stackrel{?}{=}7$; $19.4-12.4\stackrel{?}{=}7$; $7=7$

Therefore, $\boldsymbol{y=-1.6}$ and $\boldsymbol{y=4.4}$ are the real solutions to $y^2-\sqrt{8}y=7$. Furthermore, the equation $y^2-\sqrt{8}y=7$ can be factored to $\boldsymbol{(y+1.6)(y-4.4)=0}$.

5. First - Write the quadratic equation $\sqrt{5}x=2x^2$ in standard form, i.e., $2x^2-\sqrt{5}x=0$

Second - Solve the quadratic equation by choosing a solution method. $2x^2-\sqrt{5}x=0$; $x\left(2x-\sqrt{5}\right)=0$.

Therefore, the two apparent solutions are: $x=0$ and $2x-\sqrt{5}=0$; $2x=\sqrt{5}$; $x=\dfrac{\sqrt{5}}{2}$

Third - Check the answers by substituting the x values into the original equation.

I. *Let* $x=0$ *in* $\sqrt{5}x=2x^2$; $\sqrt{5}\cdot 0\stackrel{?}{=}2\cdot 0^2$; $0=0$

II. *Let* $x=\dfrac{\sqrt{5}}{2}$ *in* $\sqrt{5}x=2x^2$; $\sqrt{5}\cdot\dfrac{\sqrt{5}}{2}\stackrel{?}{=}2\cdot\left(\dfrac{\sqrt{5}}{2}\right)^2$; $\dfrac{\sqrt{5\cdot 5}}{2}\stackrel{?}{=}\cancel{2}\cdot\dfrac{5}{\cancel{4}}$; $\dfrac{\sqrt{5^2}}{2}\stackrel{?}{=}\dfrac{5}{2}$; $\dfrac{5}{2}=\dfrac{5}{2}$

Therefore, $\boldsymbol{x=0}$ and $\boldsymbol{x=\dfrac{\sqrt{5}}{2}}$ are the real solutions to $\sqrt{5}x=2x^2$. Furthermore, the equation $\sqrt{5}x=2x^2$ can be factored to $x\left(2x-\sqrt{5}\right)=0$ which is the same as $\boldsymbol{x\left(x-\dfrac{\sqrt{5}}{2}\right)=0}$.

6. First - Square both sides of the equation. $\sqrt{x^2-12}=2$; $\left(\sqrt{x^2-12}\right)^2=2^2$; $x^2-12=4$

Second - Write the quadratic equation $x^2-12=4$ in standard form, i.e., $x^2-12-4=0$; $x^2-16=0$

Third - Solve the quadratic equation by choosing a solution method. $x^2-16=0$; $(x-4)(x+4)=0$.

Therefore, the two apparent solutions are: $x-4=0$; $x=+4$ and $x-4=0$; $x=-4$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=-4$ *in* $\sqrt{x^2-12}=2$; $\sqrt{(-4)^2-12}\overset{?}{=}2$; $\sqrt{16-12}\overset{?}{=}2$; $\sqrt{4}\overset{?}{=}2$; $\sqrt{2^2}\overset{?}{=}2$; $2=2$

II. *Let* $x=4$ *in* $\sqrt{x^2-12}=2$; $\sqrt{4^2-12}\overset{?}{=}2$; $\sqrt{16-12}\overset{?}{=}2$; $\sqrt{4}\overset{?}{=}2$; $\sqrt{2^2}\overset{?}{=}2$; $2=2$

Therefore, $\boldsymbol{x=4}$ and $\boldsymbol{x=-4}$ are the real solutions to $\sqrt{x^2-12}=2$. Furthermore, the equation $\sqrt{x^2-12}=2$ can be factored to $\boldsymbol{(x+4)(x-4)=0}$.

7. First - Square both sides of the equation. $\sqrt{-8x-4}=2x+1$; $\left(\sqrt{-8x-4}\right)^2=(2x+1)^2$; $-8x-4=(2x+1)^2$

Second - Complete the square on the right hand side of the equation and simplify. $-8x-4=(2x+1)^2$

; $-8x-4=4x^2+1+4x$; $-8x-4x-4-1=4x^2$; $-12x-5=4x^2$

Third - Write the quadratic equation $-12x-5=4x^2$ in standard form, i.e., $4x^2+12x+5=0$

Fourth - Solve the quadratic equation by choosing a solution method. $4x^2+12x+5=0$; $\left(x+\frac{1}{2}\right)\left(x+\frac{5}{2}\right)=0$.

Therefore, the two apparent solutions are: $x+\frac{`1}{2}=0$; $x=-\frac{1}{2}$ and $x+\frac{5}{2}=0$; $x=-\frac{5}{2}$

Fifth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=-\frac{1}{2}$ *in* $\sqrt{-8x-4}=2x+1$; $\sqrt{-\overset{4}{\cancel{8}}\cdot\left(-\frac{1}{\cancel{2}}\right)-4}\overset{?}{=}\cancel{2}\cdot\left(-\frac{1}{\cancel{2}}\right)+1$; $\sqrt{4-4}\overset{?}{=}-1+1$; $\sqrt{0}\overset{?}{=}0$; $0=0$

II. *Let* $x=-\frac{5}{2}$ *in* $\sqrt{-8x-4}=2x+1$; $\sqrt{-\overset{4}{\cancel{8}}\cdot\left(-\frac{5}{\cancel{2}}\right)-4}\overset{?}{=}\cancel{2}\cdot\left(-\frac{5}{\cancel{2}}\right)+1$; $\sqrt{20-4}\overset{?}{=}-5+1$; $\sqrt{16}\overset{?}{=}-4$; $4\neq-4$

Therefore, the equation $\sqrt{-8x-4}=2x+1$ has one real solution, i.e., $\boldsymbol{x=-\frac{1}{2}}$.

8. First - Square both sides of the equation. $x=\sqrt{-x+2}$; $x^2=\left(\sqrt{-x+2}\right)^2$; $x^2=-x+2$

Second - Write the quadratic equation $x^2=-x+2$ in standard form, i.e., $x^2+x-2=0$

Third - Solve the quadratic equation by choosing a solution method. $x^2+x-2=0$; $(x-1)(x+2)=0$.

Therefore, the two apparent solutions are: $x-1=0$; $x=1$ and $x+2=0$; $x=-2$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=1$ *in* $x=\sqrt{-x+2}$; $1\overset{?}{=}\sqrt{-1+2}$; $1\overset{?}{=}\sqrt{1}$; $1=1$

II. *Let* $x=-2$ *in* $x=\sqrt{-x+2}$; $-2\overset{?}{=}\sqrt{-(-2)+2}$; $-2\overset{?}{=}\sqrt{2+2}$; $-2\overset{?}{=}\sqrt{4}$; $-2\overset{?}{=}\sqrt{2^2}$; $-2\neq2$

Therefore, the equation $x=\sqrt{-x+2}$ has one real solution, i.e., $\boldsymbol{x=1}$.

9. First - Square both sides of the equation. $x=\sqrt{-2x+3}$; $x^2=\left(\sqrt{-2x+3}\right)^2$; $x^2=-2x+3$

Second - Write the quadratic equation $x^2=-2x+3$ in standard form, i.e., $x^2+2x-3=0$

Third - Solve the quadratic equation by choosing a solution method. $x^2+2x-3=0$; $(x-1)(x+3)=0$.

Therefore, the two apparent solutions are: $x-1=0$; $x=1$ and $x+3=0$; $x=-3$

Fourth - Check the answers by substituting the x values into the original equation.

I. *Let* $x=1$ *in* $x=\sqrt{-2x+3}$; $1\overset{?}{=}\sqrt{-2\cdot 1+3}$; $1\overset{?}{=}\sqrt{-2+3}$; $1\overset{?}{=}\sqrt{1}$; $1=1$

II. *Let* $x=-3$ *in* $x=\sqrt{-2x+3}$; $-3\overset{?}{=}\sqrt{-2\cdot -3+3}$; $-3\overset{?}{=}\sqrt{6+3}$; $-3\overset{?}{=}\sqrt{9}$; $-3\overset{?}{=}\sqrt{3^2}$; $-3\neq 3$

Therefore, the equation $x=\sqrt{-2x+3}$ has one real solution, i.e., $\boldsymbol{x=1}$.

10. First - Square both sides of the equation. $\sqrt{x^2+3}=x+1$; $\left(\sqrt{x^2+3}\right)^2=(x+1)^2$; $x^2+3=(x+1)^2$

Second - Complete the square on the right hand side of the equation and simplify. $x^2+3=(x+1)^2$; $x^2+3=x^2+1+2x$; $x^2-x^2+3-1-2x=0$; $x^2-x^2+2-2x=0$; $2-2x=0$

Third - Solve the equation, i.e., $2-2x=0$; $2(1-x)=0$.

Therefore, the apparent solution is: $1-x=0$; $-x=-1$; $x=1$

Fourth - Check the answer by substituting the x value into the original equation.

Let $x=1$ *in* $\sqrt{x^2+3}=x+1$; $\sqrt{1^2+3}\overset{?}{=}1+1$; $\sqrt{1+3}\overset{?}{=}2$; $\sqrt{4}\overset{?}{=}2$; $\sqrt{2^2}\overset{?}{=}2$; $2=2$

Therefore, $\boldsymbol{x=1}$ is the real solution to $\sqrt{x^2+3}=x+1$.

Section 1.5 Case II Solutions - Solving Quadratic Equations Containing Fractions

1. $\frac{8}{y+1}=y-1$; $\frac{8}{y+1}=\frac{y-1}{1}$; $8\cdot 1=(y-1)\cdot(y+1)$; $8=y^2+y-y-1$; $8=y^2-1$; $8+1=y^2$; $y^2=9$; $\sqrt{y^2}=\pm\sqrt{9}$; $y=\pm\sqrt{3^2}$; $y=\pm 3$

Therefore, the two solutions are $\boldsymbol{y=+3}$ and $\boldsymbol{y=-3}$. In addition, the fractional equation $\frac{8}{y+1}=y-1$ can be expressed in factored form as $\boldsymbol{(y-3)(y+3)=0}$.

Check: I. *Let* $y=+3$ *in* $\frac{8}{y+1}=y-1$; $\frac{8}{3+1}\overset{?}{=}3-1$; $\frac{\cancel{8}^{\,2}}{\cancel{4}}\overset{?}{=}2$; $\frac{2}{1}\overset{?}{=}2$; $2=2$

II. *Let* $y=-3$ *in* $\frac{8}{y+1}=y-1$; $\frac{8}{-3+1}\overset{?}{=}-3-1$; $-\frac{\cancel{8}^{\,4}}{\cancel{2}}\overset{?}{=}-4$; $-\frac{4}{1}\overset{?}{=}-4$; $-4=-4$

2. $\frac{11x+15}{x}=-2x$; $\frac{11x+15}{x}=-\frac{2x}{1}$; $(11x+15)\cdot 1=2x\cdot x$; $11x+15=2x^2$; $2x^2-11x-15=0$; $\boldsymbol{(x+3)(2x+5)=0}$

Therefore, the two solutions are $x+3=0$; $\boldsymbol{x=-3}$ and $2x+5=0$; $2x=-5$; $x=-\frac{5}{2}$; $\boldsymbol{x=-2.5}$.

Check: I. *Let* $x=-3$ *in* $\frac{11x+15}{x}=-2x$; $\frac{11\cdot(-3)+15}{-3}\overset{?}{=}-2\cdot(-3)$; $\frac{-33+15}{-3}\overset{?}{=}6$; $\frac{-18}{-3}\overset{?}{=}6$; $\frac{\cancel{18}^{\,6}}{\cancel{3}}\overset{?}{=}6$; $\frac{6}{1}\overset{?}{=}6$; $6=6$

II. *Let* $x=-2.5$ *in* $\frac{11x+15}{x}=-2x$; $\frac{11\cdot(-2.5)+15}{-2.5}\overset{?}{=}-2\cdot(-2.5)$; $\frac{-27.5+15}{-2.5}\overset{?}{=}5$; $\frac{-12.5}{-2.5}\overset{?}{=}5$; $\frac{\cancel{12.5}^{\,5}}{\cancel{2.5}}\overset{?}{=}5$; $\frac{5}{1}\overset{?}{=}5$; $5=5$

3. $\frac{x^2}{x+3}=\frac{1}{x+3}$; $\frac{x^2}{x+3}-\frac{1}{x+3}=0$; $\frac{x^2-1}{x+3}=0$; $\frac{x^2-1}{x+3}=\frac{0}{1}$; $(x^2-1)\cdot 1=0\cdot(x+3)$; $x^2-1=0$; $x^2=1$; $\sqrt{x^2}=\pm\sqrt{1}$; $x=\pm 1$. Therefore, the two solutions are $\boldsymbol{x=+1}$ and $\boldsymbol{x=-1}$. In addition, the fractional equation $\frac{x^2}{x+3}=\frac{1}{x+3}$ can be expressed in factored form as $\boldsymbol{(x+1)(x-1)=0}$.

Check: I. *Let* $x=+1$ *in* $\frac{x^2}{x+3}=\frac{1}{x+3}$; $\frac{1^2}{1+3}\overset{?}{=}\frac{1}{1+3}$; $\frac{1}{4}=\frac{1}{4}$

II. *Let* $x=-1$ *in* $\frac{x^2}{x+3}=\frac{1}{x+3}$; $\frac{(-1)^2}{-1+3}\overset{?}{=}\frac{1}{-1+3}$; $\frac{1}{2}=\frac{1}{2}$

4. $\frac{1-2u}{u}=-u$; $\frac{1-2u}{u}=-\frac{u}{1}$; $(1-2u)\cdot 1=-u\cdot u$; $1-2u=-u^2$; $u^2-2u+1=0$; $(u-1)^2=0$; $\sqrt{(u-1)^2}=\pm\sqrt{0}$; $u-1=\pm 0$; $u=1$

Therefore, there are two repeated solution of $\boldsymbol{u=1}$. In addition, the fractional equation $\frac{1-2u}{u}=-u$ can be expressed in factored form as $\boldsymbol{(u-1)\cdot(u-1)=0}$.

Check: *Let* $u=+1$ *in* $\frac{1-2u}{u}=-u$; $\frac{1-(2\cdot 1)}{1}\overset{?}{=}-1$; $\frac{1-2}{1}\overset{?}{=}-1$; $-\frac{1}{1}\overset{?}{=}-1$; $-1=-1$

5. $x=\frac{3}{x}-2$; $x+2=\frac{3}{x}$; $\frac{x+2}{1}=\frac{3}{x}$; $(x+2)\cdot x=1\cdot 3$; $x^2+2x=3$; $x^2+2x-3=0$; $\boldsymbol{(x+3)(x-1)=0}$

Therefore, the two solutions are $x+3=0$; $\boldsymbol{x=-3}$ and $x-1=0$; $\boldsymbol{x=1}$.

Check: I. *Let* $x=-3$ *in* $x=\frac{3}{x}-2$; $-3\overset{?}{=}\frac{3}{-3}-2$; $-3\overset{?}{=}-1-2$; $-3=-3$

II. *Let* $x=1$ *in* $x=\frac{3}{x}-2$; $1\overset{?}{=}\frac{3}{1}-2$; $1\overset{?}{=}3-2$; $1=1$

6. $\frac{3x-10}{x}=-x$; $\frac{3x-10}{x}=-\frac{x}{1}$; $1\cdot(3x-10)=-x\cdot x$; $3x-10=-x^2$; $x^2+3x-10=0$; $\boldsymbol{(x+5)(x-2)=0}$

Therefore, the two solutions are $x+5=0$; $\boldsymbol{x=-5}$ and $x-2=0$; $\boldsymbol{x=2}$.

Check: I. *Let* $x=-5$ *in* $\frac{3x-10}{x}=-x$; $\frac{3\cdot(-5)-10}{-5}\overset{?}{=}-(-5)$; $\frac{-15-10}{-5}\overset{?}{=}5$; $\frac{-25}{-5}\overset{?}{=}5$; $\frac{\overset{5}{\cancel{25}}}{\cancel{5}}\overset{?}{=}5$; $\frac{5}{1}\overset{?}{=}5$; $5=5$

II. *Let* $x=2$ *in* $\frac{3x-10}{x}=-x$; $\frac{3\cdot 2-10}{2}\overset{?}{=}-2$; $\frac{6-10}{2}\overset{?}{=}-2$; $-\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\overset{?}{=}-2$; $-\frac{2}{1}\overset{?}{=}-2$; $-2=-2$

7. $u=\frac{49}{u}$; $\frac{u}{1}=\frac{49}{u}$; $u\cdot u=49\cdot 1$; $u^2=49$; $\sqrt{u^2}=\pm\sqrt{49}$; $u=\pm\sqrt{7^2}$; $u=\pm 7$

Therefore, the two solutions are $\boldsymbol{u=+7}$ and $\boldsymbol{u=-7}$. In addition, the fractional equation $u=\frac{49}{u}$ can be expressed in factored form as $\boldsymbol{(u-7)(u+7)=0}$.

Check: I. *Let* $u=+7$ *in* $u=\frac{49}{u}$; $7\overset{?}{=}\frac{\overset{7}{\cancel{49}}}{\cancel{7}}$; $7\overset{?}{=}\frac{7}{1}$; $7=7$

II. *Let* $u=-7$ *in* $u=\frac{49}{u}$; $-7\overset{?}{=}\frac{49}{-7}$; $\frac{-7}{1}\overset{?}{=}\frac{49}{-7}$; $-7\cdot -7\overset{?}{=}49\cdot 1$; $49=49$

8. $6x+17=-\frac{5}{x}$; $\frac{6x+17}{1}=-\frac{5}{x}$; $(6x+17)\cdot x=-5\cdot 1$; $6x^2+17x=-5$; $6x^2+17x+5=0$; $\boldsymbol{(2x+5)(3x+1)=0}$

Therefore, the two solutions are $2x+5=0$; $2x=-5$; $x=-\frac{5}{2}$; $\boldsymbol{x=-2.5}$ and $3x+1=0$; $3x=-1$; $x=-\frac{1}{3}$; $\boldsymbol{x=-0.333}$.

Check: I. *Let* $x=-2.5$ *in* $6x+17=-\frac{5}{x}$; $(6\cdot -2.5)+17\overset{?}{=}-\frac{5}{-2.5}$; $-15+17\overset{?}{=}\frac{5}{2.5}$; $2=2$

II. *Let* $x=-0.333$ *in* $6x+17=-\frac{5}{x}$; $(6\cdot -0.333)+17\overset{?}{=}-\frac{5}{-0.333}$; $-1.98+17\overset{?}{=}\frac{5}{0.333}$; $15.02=15.02$

9. $y+4=-\frac{3}{y}$; $\frac{y+4}{1}=-\frac{3}{y}$; $(y+4)\cdot y=-3\cdot 1$; $y^2+4y=-3$; $y^2+4y+3=0$; $\boldsymbol{(y+3)(y+1)=0}$

Therefore, the two solutions are $y+3=0$; $\boldsymbol{y=-3}$, and $y+1=0$; $\boldsymbol{y=-1}$.

Check: I. *Let* $y=-3$ *in* $y+4=-\frac{3}{y}$; $-3+4\overset{?}{=}\frac{-3}{-3}$; $1\overset{?}{=}\frac{\cancel{3}}{\cancel{3}}$; $1\overset{?}{=}\frac{1}{1}$; $1=1$

II. *Let* $y=-1$ *in* $y+4=-\frac{3}{y}$; $-1+4\overset{?}{=}\frac{-3}{-1}$; $3\overset{?}{=}\frac{3}{1}$; $3=3$

10. $3x = \frac{-5x-2}{x}$; $\frac{3x}{1} = \frac{-5x-2}{x}$; $3x \cdot x = (-5x-2) \cdot 1$; $3x^2 = -5x-2$; $3x^2 + 5x + 2 = 0$; $\mathbf{(3x+2)(x+1) = 0}$

Therefore, the two solutions are $3x+2=0$; $3x=-2$; $x=-\frac{2}{3}$; $\mathbf{x = -0.67}$ and $x+1=0$; $\mathbf{x=-1}$.

Check: I. *Let* $x=-0.67$ *in* $3x = \frac{-5x-2}{x}$; $3 \times -0.67 \overset{?}{=} \frac{(-5 \times -0.67)-2}{-0.67}$; $-0.21 \overset{?}{=} \frac{3.35-2}{-0.67}$; $-0.21 \overset{?}{=} -\frac{1.35}{0.67}$; $-0.21 = -0.21$

II. *Let* $x=-1$ *in* $3x = \frac{-5x-2}{x}$; $3 \times -1 \overset{?}{=} \frac{(-5 \times -1)-2}{-1}$; $-3 \overset{?}{=} \frac{5-2}{-1}$; $-3 \overset{?}{=} -\frac{3}{1}$; $-3=-3$

Section 1.6 Solutions - How to Choose the Best Factoring or Solution Method

1. **First Method:** (The Trial and Error Method)

Write the equation $x^2=16$ in the standard quadratic equation form $ax^2+bx+c=0$, i.e., write $x^2=16$ as $x^2+0x-16=0$. Consider the left hand side of the equation which is a polynomial. To factor the given polynomial we need to obtain two numbers whose sum is 0 and whose product is -16. Let's construct a table as follows:

Sum	***Product***
$1-1=0$	$1 \cdot (-1) = -1$
$2-2=0$	$2 \cdot (-2) = -4$
$3-3=0$	$3 \cdot (-3) = -9$
$\mathbf{4-4=0}$	$\mathbf{4 \cdot (-4) = -16}$

The last line contains the sum and the product of the two numbers that we need. Thus, $x^2=16$ or $x^2+0x-16=0$ can be factored to $\mathbf{(x-4)(x+4)=0}$

Second Method: (The Quadratic Formula Method)

First, write the equation in the standard quadratic equation form $ax^2+bx+c=0$, i.e., write $x^2=16$ as $x^2+0x-16=0$. Second, equate the coefficients of $x^2+0x-16=0$ with the standard quadratic equation by letting $a=1$, $b=0$, and $c=-16$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$; $x = \frac{-0 \pm \sqrt{0^2 - (4 \times 1 \times -16)}}{2 \times 1}$; $x = \frac{\pm\sqrt{0+64}}{2}$; $x = \frac{\pm\sqrt{64}}{2}$; $x = \pm\frac{\sqrt{8^2}}{2}$; $x = \pm\frac{8}{2}$.

Therefore, the two solutions are $\mathbf{x=-4}$ and $\mathbf{x=4}$ and the equation $x^2+0x-16=0$ can be factored to $\mathbf{(x+4)(x-4)=0}$.

Third Method: (The Square Root Property Method)

Take the square root of both sides of the equation, i.e., write $x^2=16$ as $\sqrt{x^2} = \pm\sqrt{16}$; $x = \pm\sqrt{4^2}$; $x=\pm 4$. Thus, $\mathbf{x=+4}$ and $\mathbf{x=-4}$ are the solution sets to the equation $x^2=16$ which can be represented in its factorable form as $\mathbf{(x+4)(x-4)=0}$.

Check: $(x-4)(x+4)=0$; $x \cdot x + 4 \cdot x - 4 \cdot x + 4 \cdot (-4) = 0$; $x^2+4x-4x-16=0$; $x^2+(4-4)x-16=0$; $x^2+0x-16=0$

From the above three methods using the Square Root Property method is the easiest method to use. The Trial and Error method is the second easiest method to use. Followed by the Quadratic Formula method which is the longest and somewhat a more difficult way of obtaining the factored terms.

2. **First Method:** (The Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial x^2+7x+3 we need to obtain two numbers whose sum is 7 and whose product is 3. However, after few trials, it becomes clear that such a combination of integer numbers is not possible to obtain. Therefore, **the given equation is not factorable and is referred to as PRIME**.

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2 + bx + c = 0$, equate the coefficients of $x^2 + 7x + 3 = 0$ with the standard quadratic equation by letting $a = 1$, $b = 7$, and $c = 3$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-7 \pm \sqrt{7^2 - (4 \times 1 \times 3)}}{2 \times 1}$; $x = \frac{-7 \pm \sqrt{49 - 12}}{2}$; $x = \frac{-7 \pm \sqrt{37}}{2}$; $x = \frac{-7 \pm 6.08}{2}$.

Therefore, the two solutions are $\boldsymbol{x = -6.54}$ and $\boldsymbol{x = -0.46}$ and the equation $x^2 + 7x + 3 = 0$ can be factored to $\boldsymbol{(x + 6.54)(x + 0.46) = 0}$.

Third Method: (Completing the Square Method)

$x^2 + 7x + 3 = 0$; $x^2 + 7x = -3$; $x^2 + 7x + \left(\frac{7}{2}\right)^2 = -3 + \left(\frac{7}{2}\right)^2$; $x^2 + 7x + \frac{49}{4} = -3 + \frac{49}{4}$; $\left(x + \frac{7}{2}\right)^2 = -\frac{3}{1} + \frac{49}{4}$

; $\left(x + \frac{7}{2}\right)^2 = \frac{(-3 \cdot 4) + (1 \cdot 49)}{1 \cdot 4}$; $\left(x + \frac{7}{2}\right)^2 = \frac{-12 + 49}{4}$; $\left(x + \frac{7}{2}\right)^2 = \frac{37}{4}$; $x + \frac{7}{2} = \pm\sqrt{\frac{37}{4}}$; $x + \frac{7}{2} = \pm\frac{\sqrt{37}}{2}$. Therefore, the two solutions are $\boldsymbol{x = -6.54}$ and $\boldsymbol{x = -0.46}$ and the equation $x^2 + 7x + 3 = 0$ can be factored to $\boldsymbol{(x + 6.54)(x + 0.46) = 0}$.

Check: I. *Let* $x = -0.46$ *in* $x^2 + 7x + 3 = 0$; $(-0.46)^2 + 7 \cdot (-0.46) + 3 \stackrel{?}{=} 0$; $0.2 - 3.2 + 3 \stackrel{?}{=} 0$; $-3 + 3 \stackrel{?}{=} 0$; $0 = 0$

II. *Let* $x = -6.54$ *in* $x^2 + 7x + 3 = 0$; $(-6.54)^2 + 7 \cdot (-6.54) + 3 \stackrel{?}{=} 0$; $42.8 - 45.8 + 3 \stackrel{?}{=} 0$; $42.8 - 42.8 \stackrel{?}{=} 0$; $0 = 0$

Therefore, the equation $x^2 + 7x + 3 = 0$ can be factored to $(x + 0.46)(x + 6.54) = 0$.

From the above three methods using the Quadratic Formula method may be the faster method than Completing the Square method.

3. **First Method:** (The Square Root Property Method)

$(3x + 4)^2 = 36$; $\sqrt{(3x + 4)^2} = \pm\sqrt{36}$; $3x + 4 = \pm 6$; $3x = \pm 6 - 4$; $x = \frac{\pm 6 - 4}{3}$. Thus, the two solutions are $x = \frac{6 - 4}{3}$; $\boldsymbol{x = \frac{2}{3}}$; and $x = \frac{-6 - 4}{3}$; $\boldsymbol{x = -\frac{10}{3}}$ and the equation $(3x + 4)^2 = 36$ can be factored to $\left(x - \frac{2}{3}\right)\left(x + \frac{10}{3}\right) = 0$ which is the same as $\boldsymbol{(3x - 2)(3x + 10) = 0}$.

Second Method: (The Quadratic Formula Method)

Complete the square term on the left hand side and write the equation in standard form, i.e., $(3x + 4)^2 = 36$; $9x^2 + 24x + 16 = 36$; $9x^2 + 24x + 16 - 36 = 36 - 36$; $9x^2 + 24x - 20 = 0$.

Given the standard quadratic equation $ax^2 + bx + c = 0$, equate the coefficients of $9x^2 + 24x - 20 = 0$ with the standard quadratic equation by letting $a = 9$, $b = 24$, and $c = -20$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-24 \pm \sqrt{24^2 - (4 \times 9 \times -20)}}{2 \times 9}$; $x = \frac{-24 \pm \sqrt{576 + 720}}{18}$; $x = \frac{-24 \pm \sqrt{1296}}{18}$

; $x = \frac{-24 \pm \sqrt{36^2}}{18}$; $x = \frac{-24 \pm 36}{18}$. Therefore, the two solutions are $x = \frac{-24 - 36}{18}$; $x = -\frac{\overset{10}{\cancel{60}}}{\underset{3}{\cancel{18}}}$; $\boldsymbol{x = -\frac{10}{3}}$; and $x = \frac{-24 + 36}{18}$; $x = \frac{\overset{2}{\cancel{12}}}{\underset{3}{\cancel{18}}}$; $\boldsymbol{x = \frac{2}{3}}$ and the equation $(3x + 4)^2 = 36$ can be factored to $\left(x - \frac{2}{3}\right)\left(x + \frac{10}{3}\right) = 0$ which is the same as $\boldsymbol{(3x - 2)(3x + 10) = 0}$.

Third Method: (Completing-the-Square Method)

First complete the square term on the left hand side and simplify the equation, i.e., $(3x + 4)^2 = 36$; $9x^2 + 24x + 16 = 36$; $9x^2 + 24x + 16 - 16 = 36 - -16$; $9x^2 + 24x = 20$; $\frac{\cancel{9}}{\cancel{9}}x^2 + \frac{24}{9}x = \frac{20}{9}$; $x^2 + \frac{24}{9}x = \frac{20}{9}$.

Then, complete the square in the following way:

$x^2+\frac{24}{9}x=\frac{20}{9}$; $x^2+\frac{24}{9}x+\left(\frac{\overset{4}{\cancel{24}}}{\underset{3}{\cancel{18}}}\right)^2=\frac{20}{9}+\left(\frac{\overset{4}{\cancel{24}}}{\underset{3}{\cancel{18}}}\right)^2$; $x^2+\frac{24}{9}x+\left(\frac{4}{3}\right)^2=\frac{20}{9}+\left(\frac{4}{3}\right)^2$; $x^2+\frac{24}{9}x+\frac{16}{9}=\frac{20}{9}+\frac{16}{9}$

; $\left(x+\frac{4}{3}\right)^2=\frac{20+16}{9}$; $\left(x+\frac{4}{3}\right)^2=\frac{36}{9}$; $\sqrt{\left(x+\frac{4}{3}\right)^2}=\pm\sqrt{\frac{36}{9}}$; $x+\frac{4}{3}=\pm\sqrt{\frac{6^2}{3^2}}$; $x+\frac{4}{3}=\pm\frac{6}{3}$; $x=-\frac{4}{3}\pm\frac{6}{3}$; $x=-\frac{4\pm 6}{3}$.

Therefore, the two solutions are $x=\frac{-4-6}{3}$; $\boldsymbol{x=-\frac{10}{3}}$; and $x=\frac{-4+6}{3}$; $\boldsymbol{x=\frac{2}{3}}$. In addition, the equation $(3x+4)^2=36$ can be factored to $\left(x-\frac{2}{3}\right)\left(x+\frac{10}{3}\right)=0$ which is the same as $\boldsymbol{(3x-2)(3x+10)=0}$.

Check: $(3x-2)(3x+10)=0$; $3x\cdot 3x+10\cdot 3x-2\cdot 3x-2\cdot 10=0$; $9x^2+30x-6x-20=0$; $9x^2+(30-6)x-20=0$

; $9x^2+24x-20=0$ which is the same as $(3x+4)^2=36$.

From the above three methods the Square Root Property method is the easiest method in factoring the quadratic equation, followed by the Quadratic Formula method and Completing the Square method.

4. **First Method:** (The Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial $x^2+11x+30$ we need to obtain two numbers whose sum is 11 and whose product is 30. Let's construct a table as follows:

Sum	***Product***
$1+10=11$	$1\cdot 10=10$
$2+9=11$	$2\cdot 9=18$
$3+8=11$	$3\cdot 8=24$
$4+7=11$	$4\cdot 7=28$
$\mathbf{5+6=11}$	$\mathbf{5\cdot 6=30}$

The last line contains the sum and the product of the two numbers that we need. Thus, $x^2+11x+30=0$ can be factored to $\boldsymbol{(x+5)(x+6)=0}$

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2+bx+c=0$, equate the coefficients of $x^2+11x+30=0$ with the standard quadratic equation by letting $a=1$, $b=11$, and $c=30$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-11\pm\sqrt{11^2-(4\times 1\times 30)}}{2\times 1}$; $x=\frac{-11\pm\sqrt{121-120}}{2}$; $x=\frac{-11\pm\sqrt{1}}{2}$; $x=\frac{-11\pm 1}{2}$.

Therefore, the two solutions are $x=\frac{-11+1}{2}$; $x=-\frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $\boldsymbol{x=-5}$ and $x=\frac{-11-1}{2}$; $x=-\frac{\overset{6}{\cancel{12}}}{\cancel{2}}$; $\boldsymbol{x=-6}$ and the equation $x^2+11x+30=0$ can be factored to $\boldsymbol{(x+5)(x+6)=0}$.

Third Method: (Completing-the-Square Method)

$x^2+11x+30=0$; $x^2+11x=-30$; $x^2+11x+\left(\frac{11}{2}\right)^2=-30+\left(\frac{11}{2}\right)^2$; $x^2+11x+\frac{121}{4}=-30+\frac{121}{4}$

; $\left(x+\frac{11}{2}\right)^2=-\frac{30}{1}+\frac{121}{4}$; $\left(x+\frac{11}{2}\right)^2=\frac{(-30\cdot 4)+(1\cdot 121)}{1\cdot 4}$; $\left(x+\frac{11}{2}\right)^2=\frac{-120+121}{4}$; $\left(x+\frac{11}{2}\right)^2=\frac{1}{4}$; $x+\frac{11}{2}=\pm\sqrt{\frac{1}{4}}$

; $x+\frac{11}{2}=\pm\frac{1}{2}$. Therefore, the two solutions are $\boldsymbol{x=-5}$ and $\boldsymbol{x=-6}$ and the equation $x^2+11x+30=0$ can be factored to $\boldsymbol{(x+5)(x+6)=0}$.

Check: $(x+5)(x+6)=0$; $x\cdot x+6\cdot x+5\cdot x+5\cdot 6=0$; $x^2+6x+5x+30=0$; $x^2+(6+5)x+30=0$; $x^2+11x+30=0$

From the above three methods using the Trial and Error method is the easiest method to obtain the factored terms. Completing the Square method is the second easiest method to use, followed by the Quadratic Formula method which is the longest and perhaps the most difficult way of obtaining the factored terms.

5. **First Method:** (The Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial $5t^2+4t-1$ we need to obtain two numbers whose sum is 4 and whose product is $5\cdot -1=-5$. Let's construct a table as follows:

Sum	***Product***
$8-4=4$	$8\cdot(-4)=-32$
$7-3=4$	$7\cdot(-3)=-21$
$6-2=4$	$6\cdot(-2)=-12$
$\mathbf{5-1=4}$	$\mathbf{5\cdot(-1)=-5}$

The last line contains the sum and the product of the two numbers that we need. Thus, $5t^2+4t-1=0$; $5t^2+(5-1)t-1=0$; $5t^2+5t-t-1=0$; $5t(t+1)-(t+1)=0$; $\mathbf{(t+1)(5t-1)=0}$

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $at^2+bt+c=0$, equate the coefficients of $5t^2+4t-1=0$ with the standard quadratic equation by letting $a=5$, $b=4$, and $c=-1$. Then,

Given: $t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$; $t=\dfrac{-4\pm\sqrt{4^2-(4\times5\times-1)}}{2\times5}$; $t=\dfrac{-4\pm\sqrt{16+20}}{10}$; $t=\dfrac{-4\pm\sqrt{36}}{10}$; $t=\dfrac{-4\pm\sqrt{6^2}}{6}10$; $t=\dfrac{-4\pm6}{10}$. Therefore, the two solutions are $\mathbf{t=-1}$ and $\mathbf{t=\frac{1}{5}}$ and the equation $5t^2+4t-1=0$ can be factored to $(t+1)\left(t-\frac{1}{5}\right)=0$ which is the same as $\mathbf{(t+1)(5t-1)=0}$.

Third Method: (Completing-the-Square Method)

$5t^2+4t-1=0$; $5t^2+4t=1$; $\frac{\cancel{5}}{\cancel{5}}t^2+\frac{4}{5}t=\frac{1}{5}$; $t^2+\frac{4}{5}t=\frac{1}{5}$; $t^2+\frac{4}{5}t+\left(\dfrac{\frac{\cancel{4}\,2}{\cancel{10}\,5}}{}\right)^2=\frac{1}{5}+\left(\dfrac{\frac{\cancel{4}\,2}{\cancel{10}\,5}}{}\right)^2$; $t^2+\frac{4}{5}t+\left(\frac{2}{5}\right)^2=\frac{1}{5}+\left(\frac{2}{5}\right)^2$; $t^2+\frac{4}{5}t+\frac{4}{25}=\frac{1}{5}+\frac{4}{25}$; $\left(t+\frac{2}{5}\right)^2=\frac{(1\cdot25)+(4\cdot5)}{5\cdot25}$; $\left(t+\frac{2}{5}\right)^2=\frac{25+20}{125}$; $\left(t+\frac{2}{5}\right)^2=\dfrac{\overset{9}{\cancel{45}}}{\underset{25}{\cancel{125}}}$; $\left(t+\frac{2}{5}\right)^2=\frac{9}{25}$; $\sqrt{\left(t+\frac{2}{5}\right)^2}=\pm\sqrt{\frac{9}{25}}$; $t+\frac{2}{5}=\pm\sqrt{\frac{3^2}{5^2}}$; $t+\frac{2}{5}=\pm\frac{3}{5}$; $t=-\frac{2}{5}\pm\frac{3}{5}$. Therefore, the two solutions are $t=-\frac{2}{5}-\frac{3}{5}$; $t=\frac{-2-3}{5}$; $t=-\frac{5}{5}$; $\mathbf{t=-1}$ and $t=-\frac{2}{5}+\frac{3}{5}$; $t=\frac{-2+3}{5}$; $\mathbf{t=\frac{1}{5}}$ and the equation $5t^2+4t-1=0$ can be factored to $(t+1)\left(t-\frac{1}{5}\right)=0$ which is the same as $\mathbf{(t+1)(5t-1)=0}$.

Check: I. *Let* $t=\frac{1}{5}$ *in* $5t^2+4t-1=0$; $5\cdot\left(\frac{1}{5}\right)^2+4\cdot\left(\frac{1}{5}\right)-1\overset{?}{=}0$; $\frac{\cancel{5}}{\underset{5}{\cancel{25}}}+\frac{4}{5}-1\overset{?}{=}0$; $\frac{1}{5}+\frac{4}{5}-1\overset{?}{=}0$; $\frac{1+4}{5}-1\overset{?}{=}0$; $\frac{\cancel{5}}{\cancel{5}}-1\overset{?}{=}0$; $1-1\overset{?}{=}0$; $0=0$

II. *Let* $t=-1$ *in* $5t^2+4t-1=0$; $5\cdot(-1)^2+4\cdot(-1)-1\overset{?}{=}0$; $5-4-1\overset{?}{=}0$; $5-5\overset{?}{=}0$; $0=0$

From the above three methods using the Trial and Error method is the easiest method to obtain the factored terms. The Quadratic Formula method is the second easiest method to use, followed by Completing-the-Square method.

6. **First Method:** (The Trial and Error Method)

To apply the Trial and Error method to the equation $(2x+6)^2=36$ we need to complete and simplify the square in the left hand side of the equation, i.e., $(2x+6)^2=36$; $4x^2+36+24x=36$; $4x^2+24x+36-36=36-36$; $4x^2+24x+0=0$

; $\frac{\cancel{4}}{\cancel{4}}x^2 + \frac{\overset{6}{\cancel{24}}}{\cancel{4}}x + 0 = 0$; $x^2 + 6x + 0 = 0$. Consider the left hand side of the equation which is a polynomial. To factor the polynomial $x^2 + 6x + 0$ we need to obtain two numbers whose sum is 6 and whose product is $6 \cdot 0 = 0$. Let's construct a table as follows:

Sum	***Product***
$4 + 2 = 6$	$4 \cdot 2 = 8$
$5 + 1 = 6$	$5 \cdot 1 = 5$
$\mathbf{6 + 0 = 6}$	$\mathbf{6 \cdot 0 = 0}$

The last line contains the sum and the product of the two numbers that we need. Thus, $(2x + 6)^2 = 36$ can be factored to $(x + 0)(x + 6) = 0$ which is the same as $\boldsymbol{x(x + 6) = 0}$

Second Method: (The Greatest Common Factoring Method)

First complete the square term on the left hand side and simplify the equation:

$(2x + 6)^2 = 36$; $4x^2 + 24x + 36 = 36$; $4x^2 + 24x = 36 - 36$; $4x^2 + 24x = 0$; $\frac{\cancel{4}}{\cancel{4}}x^2 + \frac{\overset{6}{\cancel{24}}}{\cancel{4}}x = \frac{0}{4}$; $x^2 + 6x = 0$

Then, Factor out the greatest common monomial term x .

$x^2 + 6x = 0$; $x(x + 6) = 0$. Thus, the two solution to the equation are: $\boldsymbol{x = 0}$ and $x + 6 = 0$; $\boldsymbol{x = -6}$

Hence, the equation $(2x + 6)^2 = 36$ can be factored to $(x + 0)(x + 6) = 0$ which is the same as $\boldsymbol{x(x + 6) = 0}$.

Third Method: (The Square Root Property Method)

$(2x + 6)^2 = 36$; $\sqrt{(2x + 6)^2} = \pm\sqrt{36}$; $2x + 6 = \pm 6$; $2x = -6 \pm 6$. Thus, the two solutions are $2x = -6 + 6$; $2x = 0$; $\frac{\cancel{2}}{\cancel{2}}x = \frac{0}{2}$; $\boldsymbol{x = 0}$ and $2x = -6 - 6$; $2x = -12$; $\frac{\cancel{2}}{\cancel{2}}x = -\frac{\overset{6}{\cancel{12}}}{\cancel{2}}$; $\boldsymbol{x = -6}$ and the equation $(2x + 6)^2 = 36$ can be factored to $(x + 0)(x + 6) = 0$ which is the same as $\boldsymbol{x(x + 6) = 0}$.

Check: I. *Let* $x = 0$ *in* $(2x + 6)^2 = 36$; $[(2 \cdot 0) + 6]^2 \overset{?}{=} 36$; $6^2 \overset{?}{=} 36$; $36 = 36$

II. *Let* $x = -6$ *in* $(2x + 6)^2 = 36$; $[(2 \cdot -6) + 6]^2 \overset{?}{=} 36$; $(-12 + 6)^2 \overset{?}{=} 36$; $(-6)^2 \overset{?}{=} 36$; $36 = 36$

From the above three methods the Greatest Common Factoring method is the easiest method. The Square Root Property method is the second easiest, followed by the Trail and Error method.

7. **First Method:** (The Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial $y^2 - 8y + 15$ we need to obtain two numbers whose sum is -8 and whose product is 15 . Let's construct a table as follows:

Sum	***Product***
$-4 - 4 = -8$	$-4 \cdot -4 = 16$
$\mathbf{-5 - 3 = -8}$	$\mathbf{-5 \cdot -3 = 15}$
$-6 - 2 = -8$	$-6 \cdot -2 = 12$
$-7 - 1 = -8$	$-7 \cdot -1 = 7$

The second line contains the sum and the product of the two numbers that we need. Thus, $y^2 - 8y + 15 = 0$ can be factored to $\boldsymbol{(y - 3)(y - 5) = 0}$.

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2 + bx + c = 0$, equate the coefficients of $y^2 - 8y + 15 = 0$ with the standard quadratic equation by letting $a = 1$, $b = -8$, and $c = 15$. Then,

Given: $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $y = \frac{-(-8) \pm \sqrt{(-8)^2 - (4 \times 1 \times 15)}}{2 \times 1}$; $y = \frac{8 \pm \sqrt{64 - 60}}{2 \times 1}$; $y = \frac{8 \pm \sqrt{4}}{2}$; $y = \frac{8 \pm \sqrt{2^2}}{2}$

; $y = \frac{8 \pm 2}{2}$. Therefore, the two solutions are $y = \frac{8-2}{2}$; $y = \frac{\cancel{6}}{\cancel{2}}$; $\boldsymbol{y = 3}$ and $y = \frac{8+2}{2}$; $y = \frac{\overset{5}{\cancel{10}}}{\cancel{2}}$; $\boldsymbol{y = 5}$ and the equation $y^2 - 8y + 15 = 0$ can be factored to $\boldsymbol{(y-3)(y-5) = 0}$.

Third Method: (Completing-the-Square Method)

$$y^2 - 8y + 15 = 0 \; ; \; y^2 - 8y = -15 \; ; \; y^2 - 8y + \left(-\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2 = -15 + \left(-\frac{\overset{4}{\cancel{8}}}{\cancel{2}}\right)^2 \; ; \; y^2 - 8y + \left(-\frac{4}{1}\right)^2 = -15 + \left(-\frac{4}{1}\right)^2$$

; $y^2 - 8y + 16 = -15 + 16$; $y^2 - 8y + 16 = 1$; $(y-4)^2 = 1$; $\sqrt{(y-4)^2} = \pm\sqrt{1}$; $y - 4 = \pm 1$; $y = 4 \pm 1$. Therefore, the two solutions are $\boldsymbol{y = 3}$ and $\boldsymbol{y = 5}$ and the equation $y^2 - 8y + 15 = 0$ can be factored to $\boldsymbol{(y-3)(y-5) = 0}$.

Check: $(y-3)(y-5) = 0$; $y \cdot y - 5 \cdot y - 3 \cdot y + (-3) \cdot (-5) = 0$; $y^2 - 5y - 3y + 15 = 0$; $y^2 + (-5-3)y + 15 = 0$; $y^2 - 8y + 15 = 0$

From the above three methods using the Trial and Error method is the easiest method to obtain the factored terms. Completing-the-Square method is the second easiest method to use, followed by the Quadratic Formula method which is the longest and perhaps the most difficult way of obtaining the factored terms.

8. **First Method:** (The Trial and Error Method)

Write the equation $w^2 = -7$ in the standard quadratic equation form $aw^2 + bw + c = 0$, i.e., write $w^2 = -7$ as $w^2 + 0w + 7 = 0$. Consider the left hand side of the equation which is a polynomial. To factor the polynomial $w^2 + 0w + 7$ we need to obtain two numbers whose sum is 0 and whose product is 7. Let's construct a table as follows:

Sum	***Product***
$1 - 1 = 0$	$1 \cdot (-1) = -1$
$2 - 2 = 0$	$2 \cdot (-2) = -4$
$3 - 3 = 0$	$3 \cdot (-3) = -9$
$4 - 4 = 0$	$4 \cdot (-4) = -16$

After several trials it becomes clear that the given equation can not be simplified using the Trail and Error method. Therefore, **the given equation is not factorable and is referred to as PRIME.**

Second Method: (The Quadratic Formula Method)

First, write the equation in the standard quadratic equation form $aw^2 + bw + c = 0$, i.e., write $w^2 = -7$ as $w^2 + 0w + 7 = 0$. Second, equate the coefficients of $w^2 + 0w + 7 = 0$ with the standard quadratic equation by letting $a = 1$,, $b = 0$, and $c = 7$. Then,

Given: $w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $w = \frac{-0 \pm \sqrt{0^2 - (4 \times 1 \times 7)}}{2 \times 1}$; $w = \frac{\pm\sqrt{0 - 28}}{2}$; $w = \frac{\pm\sqrt{-28}}{2}$. **However, since the number under the radical is a negative number, the given equation has no real solution and can not be factored.**

Third Method: (The Square Root Property Method)

Take the square root of both sides of the equation, i.e., write $w^2 = -7$ as $\sqrt{w^2} = \pm\sqrt{-7}$; $w = \pm\sqrt{-7}$

Again, since the number under the radical is negative, the given equation has no real solution and can not be factored.

9. **First Method:** (The Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial $6x^2 + x - 1$ we need to obtain two numbers whose sum is 1 and whose product is $6 \cdot -1 = -6$. Let's construct a table as follows:

Sum	***Product***
$5 - 4 = 1$	$5 \cdot -4 = -20$
$4 - 3 = 1$	$4 \cdot -3 = -12$
$\boldsymbol{3 - 2 = 1}$	$\boldsymbol{2 \cdot -3 = -6}$
$2 - 1 = 1$	$2 \cdot -1 = -2$

The third line contains the sum and the product of the two numbers that we need. Therefore, $6x^2+x-1=0$; $6x^2+(3-2)x-1=0$; $6x^2+3x-2x-1=0$; $3x(2x+1)-(2x+1)=0$; $\mathbf{(2x+1)(3x-1)=0}$

Second Method: (The Quadratic Formula Method)

Given the standard quadratic equation $ax^2+bx+c=0$, equate the coefficients of $6x^2+x-1=0$ with the standard quadratic equation by letting $a=6$, $b=1$, and $c=-1$. Then,

Given: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-1\pm\sqrt{1^2-(4\times6\times-1)}}{2\times6}$; $x=\frac{-1\pm\sqrt{1+24}}{12}$; $x=\frac{-1\pm\sqrt{25}}{12}$; $x=\frac{-1\pm\sqrt{5^2}}{12}$; $x=\frac{-1\pm5}{12}$. Therefore, the two solutions are $x=\frac{-1-5}{12}$; $x=-\frac{\cancel{6}}{\underset{2}{\cancel{12}}}$; $\mathbf{x=-\frac{1}{2}}$ and $x=\frac{-1+5}{12}$; $x=\frac{\cancel{4}}{\underset{3}{\cancel{12}}}$; $\mathbf{x=\frac{1}{3}}$ and the equation $6x^2+x-1=0$ can be factored to $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{3}\right)=0$ which is the same as $\mathbf{(2x+1)(3x-1)=0}$.

Third Method: (Completing-the-Square Method)

$6x^2+x-1=0$; $6x^2+x=1$; $\frac{\cancel{6}}{\cancel{6}}x^2+\frac{1}{6}x=\frac{1}{6}$; $x^2+\frac{1}{6}x=\frac{1}{6}$; $x^2+\frac{1}{6}x+\left(\frac{1}{12}\right)^2=\frac{1}{6}+\left(\frac{1}{12}\right)^2$

; $x^2+\frac{1}{6}x+\frac{1}{144}=\frac{1}{6}+\frac{1}{144}$; $\left(x+\frac{1}{12}\right)^2=\frac{(1\cdot144)+(1\cdot6)}{6\cdot144}$; $\left(x+\frac{1}{12}\right)^2=\frac{144+6}{864}$; $\left(x+\frac{1}{12}\right)^2=\frac{\overset{75}{\cancel{150}}}{\underset{432}{\cancel{864}}}$; $\left(x+\frac{1}{12}\right)^2=\frac{75}{432}$

; $\sqrt{\left(x+\frac{1}{12}\right)^2}=\pm\sqrt{\frac{75}{432}}$; $x+\frac{1}{12}=\pm\sqrt{0.174}$; $x+0.083=\pm0.417$; $x=-0.083\pm0.417$.

Therefore, the two solutions are $x=-0.083-0.417$; $x=-0.5$; $\mathbf{x=-\frac{1}{2}}$ and $x=-0.083+0.417$; $x=0.33$; $\mathbf{x=\frac{1}{3}}$ and the equation $6x^2+x-1=0$ can be factored to $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{3}\right)=0$ which is the same as $\mathbf{(2x+1)(3x-1)=0}$.

Check: I. *Let* $t=\frac{1}{3}$ *in* $6x^2+x-1=0$; $6\cdot\left(\frac{1}{3}\right)^2+\frac{1}{3}-1\overset{?}{=}0$; $\overset{2}{\cancel{6}}\cdot\frac{1}{\underset{3}{\cancel{9}}}+\frac{1}{3}-1\overset{?}{=}0$; $\frac{2}{3}+\frac{1}{3}-1\overset{?}{=}0$; $\frac{2+1}{3}-1\overset{?}{=}0$; $\frac{3}{3}-1\overset{?}{=}0$

; $1-1\overset{?}{=}0$; $0=0$

II. *Let* $t=-\frac{1}{2}$ *in* $6x^2+x-1=0$; $6\cdot\left(-\frac{1}{2}\right)^2-\frac{1}{2}-1\overset{?}{=}0$; $\overset{3}{\cancel{6}}\cdot\frac{1}{\underset{2}{\cancel{4}}}-\frac{1}{2}-1\overset{?}{=}0$; $\frac{3}{2}-\frac{1}{2}-1\overset{?}{=}0$; $\frac{3-1}{2}-1\overset{?}{=}0$

; $\frac{2}{2}-1\overset{?}{=}0$; $1-1\overset{?}{=}0$; $0=0$

From the above three methods using the Quadratic Formula method is the easiest method to obtain the factored terms. The Trial and Error method is the second easiest method to use, followed by Completing-the-Square method.

10. **First Method:** (The Perfect Square Approach)

First - Check and see if the coefficients of the first and the last term are perfect squares, i.e., x^2-4x+4 ; x^2-4x+2^2

Second - Check and see if by multiplying the base of the last term by $-2x$ we can obtain the middle term, i.e., $2\cdot-2x=-4x$.

Third - Since $-4x$ is the same as the middle term of the given polynomial therefore, the trinomial can be written as:

$x^2-4x+2^2=(x-2)^2=\mathbf{(x-2)(x-2)}$

Second Method: (The Quadratic Formula Method)

Based on our earlier stated assumption, we write the given trinomial as a quadratic equation, i.e., $x^2-4x+4=0$ and apply the Quadratic Formula by letting $a=1$, $b=-4$, and $c=4$. Then,

Given: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times 4)}}{2 \times 1}$; $x = \frac{4 \pm \sqrt{16 - 16}}{2}$; $x = \frac{4 \pm \sqrt{0}}{2}$; $x = \frac{4 \pm 0}{2}$; $x = \frac{\overset{2}{\cancel{4}}}{\cancel{2}}$;

$x = 2$. Therefore, the quadratic equation has two repeated solution, i.e., $\boldsymbol{x = 2}$ and $\boldsymbol{x = 2}$ and the equation $x^2 - 4x + 4 = 0$ can be factored to $\boldsymbol{(x-2)(x-2) = 0}$.

Third Method: (Trial and Error Method)

Consider the left hand side of the equation which is a polynomial. To factor the polynomial $x^2 - 4x + 4$ we need to obtain two numbers whose sum is -4 and whose product is 4 . Let's construct a table as follows:

Sum	***Product***
$-6 + 2 = -4$	$-6 \cdot 2 = -12$
$-5 + 1 = -4$	$-5 \cdot 1 = -5$
$-3 - 1 = -4$	$-3 \cdot -1 = 3$
$\boldsymbol{-2 - 2 = -4}$	$\boldsymbol{-2 \cdot -2 = 4}$

The last line contains the sum and the product of the two numbers that we need. Thus, $x^2 - 4x + 4 = 0$ can be factored to $\boldsymbol{(x-2)(x-2) = 0}$.

Check: $(x-2)(x-2) = 0$; $x \cdot x - 2 \cdot x - 2 \cdot x + (-2) \cdot (-2) = 0$; $x^2 - 2x - 2x + 4 = 0$; $x^2 + (-2-2)x + 4 = 0$

; $x^2 - 4x + 4 = 0$

From the above three methods the Perfect Square method is the easiest method in factoring the trinomial followed by the Trial and Error method and the Quadratic Formula method.

About the Author

Dan Hamilton received his B.S. degree in Electrical Engineering from Oklahoma State University and Master's degree, also in Electrical Engineering from the University of Texas at Austin. He has taught a number of math and engineering courses as a visiting lecturer at the University of Oklahoma, Department of Mathematics, and as a faculty member at Rose State College, Department of Engineering Technology, at Midwest City, Oklahoma. He is currently working in the field of aerospace technology and has published several books and numerous technical papers.